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Stochastic approach to domain wall dynamics and ferromagnetic hysteresis

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Abstract

We propose a set of stochastic equations to describe domain wall (DW) dynamics in a random pinning field. The pinning field is characterized by two correlation lengths, controlling its stationarity and the reversible permeability. Computer simulations of the hysteresis loop shapes are investigated as a function of the ratio of the correlation lengths.

Keywords: Domain wall dynamics; Hysteresis; Stochastic processes

The Barkhausen effect (BE) is considered one of the basic fingerprints of the magnetization process. In previous works, the physical interpretation of the BE was investigated using a theory successfully applied to the description of various experimental data, such as power spectra, probability distribution of DW velocity [1], scaling and fractal properties of Barkhausen jumps [2], and the dependence of the coercivity on the demagnetizing field [3]. In this paper, we apply the same approach to the description of hysteresis loops. Our model is similar to that proposed by Néel [4], but it has a richer mathematical structure.

The dynamics of magnetization is described in terms of a single degree of freedom, Φ, evolving under the action of external field $H_a(t)$. Φ is the magnetic flux through a given cross section, S, of the slab- shaped magnetic specimen. The equation for the Φ dynamics [1] is:

$$\sigma G\dot{\Phi} = H_a - \Phi/S\mu - H_p, \qquad (1)$$

where σ is the electrical conductivity, $G \approx 0.1356$, and $\dot{\Phi}$ is the magnetic flux rate of change. $\Phi/S\mu$ represents the magnetostatic field, under the assumption of constant permeability μ. H_p is the pinning field experienced by the DW, arising from the structural disorder, and from the interactions with other DWs. It is assumed to be a random function of Φ. In the simplest case, $H_p(\Phi) = W(\Phi)$, where W is the Wiener–Lévy process. This description proved to be useful in the study of the Barkhausen effect [1,2]. Yet, when applied to the interpretation of hysteresis loop shapes, the assumption $H_p(\Phi) = W(\Phi)$ has two quite undesirable properties: (i) H_p is non-stationary; and (ii) H_p is non-differentiable at any point. These drawbacks can be eliminated by introducing two correlation lengths, say ξ_2

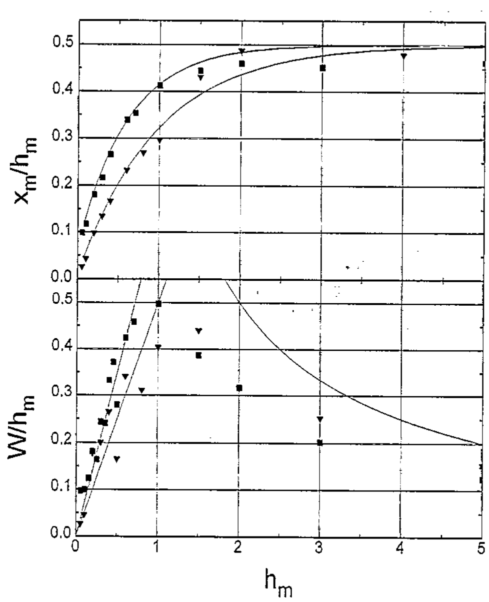

Fig. 1. Permeability data for $\beta = 2$, $\alpha = 0.01$ (■) and $\alpha = 0.0001$ (▼) fitted by Eq. (5); hysteresis loss W/h_m^2 for the same parameters fitted by straight lines $\frac{4}{3}bh_m$ at low fields; the hyperbolic law $1/h_m$ for $\alpha \to 0$ is also shown.

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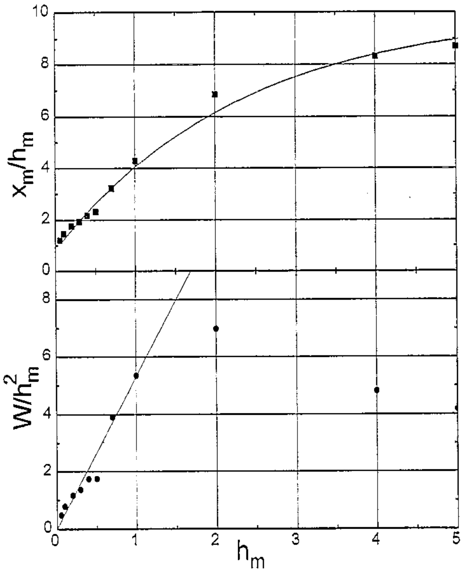

Fig. 2. Permeability data for $\beta = 0.1$, $\alpha = 1$ fitted by Eq. (5); hysteresis loss W/h_m^2 for the same parameters fitted by the straight line $\frac{4}{3}bh_\mathrm{m}$ at low fields.

and $\xi_1 = \alpha\xi_2$ ($\alpha \le 1$), such that H_p is stationary for $\Delta\Phi \gg \xi_2$ and a well defined permeability exists for $\Delta\Phi \ll \xi_1$. With the introduction of ξ_2, it is convenient to express the variance of the Wiener–Lévy process as $\langle |\mathrm{d}W|^2 \rangle = (2A_\mathrm{H}^2/\xi_2)\mathrm{d}\Phi$, where A_H measures the scale of the pinning field variations. This leads to a set of equations (in dimensionless form):

$$\beta v = h_\mathrm{a}(u) - \beta x - h_\mathrm{p}(u), \tag{2}$$

$$\frac{\mathrm{d}h_\mathrm{p}}{\mathrm{d}x} = s - h_\mathrm{p}, \tag{3}$$

$$\frac{\mathrm{d}s}{\mathrm{d}x} + \frac{s}{\alpha} + \frac{1}{\alpha}\frac{\mathrm{d}w}{\mathrm{d}x}, \tag{4}$$

where $\langle |\mathrm{d}w|^2 \rangle = 2\mathrm{d}x$, $h_\mathrm{a} = H_\mathrm{a}/A_\mathrm{H}$, $h_\mathrm{p} = H_\mathrm{p}/A_\mathrm{H}$, $w = W/A_\mathrm{H}$ and $u = t/\tau$, with $\tau = \sigma GS\mu$, $x = \Phi/\xi_2$ and $v = \mathrm{d}x/\mathrm{d}u$. β is defined as $\beta = \xi_2/(A_\mathrm{H}S\mu)$. Finally, $s = S_\mathrm{H}\xi_2/A_\mathrm{H}$, where S_H controls the H_p slope for $\Delta\Phi \ll \xi_1$, i.e. $\Delta x \ll \alpha$.

As a first step, we faced the problem by numerically simulating the behavior of the system. Three different cases, A, B and C, defined in terms of the parameters (α,β) were studied: case A (1,0.1); case B (10^{-2},2); and case C (10^{-4},2). Figs. 1 and 2 show the h_m dependence of the permeability $x_\mathrm{m}/h_\mathrm{m}$ (x_m is the loop peak displacement) and of the loop area W (represented as W/h_m^2). At high fields, the permeability always tends to the constant value $1/\beta$, as expected from Eq. (2), which, on average, just reduces to $h_\mathrm{a} - \beta x = 0$. At field amplitudes $h_\mathrm{m} \le 1/\beta$, the behavior is in agreement with the Rayleigh law $x_\mathrm{m}/h_\mathrm{m} = a + bh_\mathrm{m}$. The simplest way to describe both regimes is:

$$\frac{x_\mathrm{m}}{h_\mathrm{m}} = \frac{1}{\beta} - (1/\beta - a)\exp\left(-\frac{bh_\mathrm{m}}{1/\beta - a}\right). \tag{5}$$

By fitting the simulation data with Eq. (5), we obtained the parameters (a,b): case A (0.85,4); case B (0.07,0.7); case C (0.01,0.5). Case A, where $\xi_1 = \xi_2$, is expected to be the closest to Néel's model, which is characterised by a random pinning field profile where the correlation in the pinning field values and slopes is lost on the same scale. Néel's model predicts $a = 0.81$ and $b = 1/\pi$. Our value for a is in good agreement with Neel's, while b is about one order of magnitude greater. This disagreement arises from the fact that Néel considered a DW initially in the demagnetized state. Conversely, in our simulations we simply placed the DW in a random stable position: a choice which inevitably leads to higher dissipation and thus to higher b values. The new regime $\alpha \ll 1$ is not present in Néel's model. It can be seen that the values of a approximately scale as $\sqrt{\alpha}$. In fact, the DW reversible permeability is controlled by the local pinning field slope. This slope is of the order of $1/s \approx \sqrt{\alpha}$. The simulations also show that $b \to 0.5$ for $\alpha \to 0$. In fact, it can be proved that, in the limit $\alpha \to 0$, the DW behavior is equivalent to a Preisach distribution [5] $p(h_\mathrm{c},h_\mathrm{u}) = \frac{1}{2}\exp(-\beta h_\mathrm{c})$. It can be shown that the value of p at $h_\mathrm{c} = 0$ is just the value of the parameter b of the Rayleigh law, which gives $b = 0.5$. Figs. 1 and 2 also show the behavior of the hysteresis loss per cycle, W. In the cases where the Rayleigh law holds, one expects $W = \frac{4}{3}bh_\mathrm{m}^3$. The straight lines in the figures show that this is indeed the case. At larger fields, when the magnetostatic field βx becomes dominant, the hysteresis loop attains a constant half-width, h_0, independent of h_m. In this limit, $W = 4h_0x_\mathrm{m} = 4h_0h_\mathrm{m}/\beta$ and $W/h_\mathrm{m}^2 = 4h_0/\beta h_\mathrm{m}$. In Ref. [3], it is shown that, when $\alpha \to 0$, $h_0 = 1/\beta$. This represents an upper limit for the loss behavior, as shown in Fig. 1 ($W/h_\mathrm{m}^2 = 4/\beta^2 h_\mathrm{m} = 1/h_\mathrm{m}$ for $\beta = 2$). The difference between this line and the values obtained from the simulations is the effect of the finite value of α [3].

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References

[1] B. Alessandro, C. Beatrice, G. Bertotti and A. Montorsi, J. Appl. Phys. 68 (1990) 2901; 68 (1990) 2908.
[2] G. Durin, A. Magni and G. Bertotti, J. Magn. Magn. Mater. 140–144 (1995) 1835.
[3] L. Pust and G. Bertotti, IEEE Trans. Magn. 30 (1994) 834.
[4] L. Néel, Cah. Phys. 12 (1942) 1; 13 (1943) 18.
[5] G. Bertotti, V. Basso and G. Durin, Presented at MMM Conf., Philadelphia, 1995.