# Scaling aspects of domain wall dynamics and Barkhausen effect in ferromagnetic materials

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It is shown through theoretical considerations on stochastic domain wall motion in a perturbed medium with quenched-in disorder that the Barkhausen signal $v$, as well as the size $\Delta x$ and duration $\Delta u$ of Barkhausen jumps follow scaling distributions of the form $v^{-\alpha}$, $\Delta x^{-\beta}$, $\Delta u^{-\gamma}$, where $\alpha=1-c$, $\beta=3/2-c/2$, $\gamma=2-c$, and $c$ is proportional to the magnetization rate. In order to test these predictions, Barkhausen effect experiments were performed on polycrystalline SiFe alloys. Preliminary experiments to determine both the absolute value and the $c$ dependence of the measured exponents are in agreement with the theoretical predictions.

## I. INTRODUCTION

There have been widespread attempts in the past to describe the most evident feature of the Barkhausen Effect (BE), the existence of Barkhausen jumps (BJ), in terms of clustering of elementary domain wall (DW) displacements triggered by some local instability.[1,2] More recently, there has been renewed interest in BE scaling properties,[3] and in their connection with so-called self-organized criticality.[4] These studies have led to experimental estimates of the scaling exponents describing the distribution of BJ size and duration, but the derivation of these exponents from physical models is still at a preliminary stage.

In this paper we discuss BE scaling properties in the frame of the Langevin approach developed in Ref. 5, where the BE is associated with stochastic DW dynamics in a medium with quenched-in disorder. This description leads to two basic results: (i) there exists a dimensionless parameter $c$, such that the DW motion has a jerky character and proceeds by BJs when $c<1$, whereas it is continuous when $c>1$; (ii) when $c<1$, there exist BJs widely distributed in size and duration, and characterized by self-similar properties. In this paper, we show that, in the $c<1$ regime, the distributions of BJ size $\Delta x$ and duration $\Delta u$ follow scaling laws of the form $(\Delta x)^{-\beta}$ and $(\Delta u)^{-\gamma}$, with $\beta=3/2-c/2$ and $\gamma=2-c$.

These predictions, which are in qualitative agreement with recent data,[3,4] were tested through BE measurements under controlled values of permeability and magnetization rate, whereby independent estimates of $\beta$, $\gamma$, and $c$ could be obtained. This permitted a test of the theory with no adjustable parameters.

## II. MODEL

In Refs. 5 and 6, DW dynamics is described in terms of a single degree of freedom subject to viscous-like friction (i.e., Joule dissipation through eddy currents). This leads to an equation of the form $v_{\rm DW}\propto H_a-kx_{\rm DW}-H_p(x_{\rm DW})$, where $v_{\rm DW}$ is the DW velocity, $x_{\rm DW}$ is the DW position, $H_a$ is the applied field, $kx_{\rm DW}$ is the restoring force due to magnetostatic effects, and $H_p(x_{\rm DW})$, describing DW pinning interactions, is a random function of the DW position. By taking the time derivative of the previous relation and by assuming, as is commonly the case in BE experiments, that the applied field increases in time at a constant rate, we obtain, in terms of convenient dimensionless variables [$u$ for time, $x$ and $v=dx/du$ for the DW position and velocity, $h_p(x)$ for applied and pinning fields], the equation

$$\frac{dv}{du}+(v-c)=-\frac{dh_p}{du}. \tag{1}$$

Quantitative predictions are worked out by making specific assumptions on the random process $h_p(x)$. The properties of $h_p(x)$ have been experimentally investigated[7] for some special systems containing a single active DW. It was found that $h_p(x)$ can be approximately described by the Wiener–Lévy (WL) process, i.e., $h_p(x)$ is a process with independent increments $dh_p$ characterized by $\langle dh_p\rangle=0$, $\langle|dh_p|^2\rangle=2\,dx=2v\,du$. Under this assumption, the problem can be solved considering the Fokker–Planck equation for the conditional probability density $P(v,u|v_0)$:[5]

$$\frac{\partial P}{\partial u}-\frac{\partial}{\partial v}\left((v-c+1)P+v\,\frac{\partial P}{\partial v}\right)=0. \tag{2}$$

In particular, the stationary amplitude distribution $P_u(v)$ is given by

$$P_u(v)=\frac{1}{\Gamma(c)}\,v^{c-1}\exp(-v) \tag{3}$$

According to Eq. (3), the behavior of $v(u)$ changes drastically when $c$ crosses the value $c=1$. Computer simulations[5] show that, for $c<1$, $v(u)$ is made of a random sequence of BJs widely distributed in size and duration. The power-law divergence in $P_u(v)$ suggests the existence of scaling properties in the distribution of such BJs. In order to clarify this point, let us consider the region $v\ll1$, where the $v$ term in the expression $(v-c+1)$ of Eq. (2) is negligible and Eq. (3) becomes $P_u(v)\sim v^{c-1}$. This also corresponds to neglecting the $v$ term of Eq. (1), i.e., $dv/du-c\cong-dh_p/du$. This approximate equation describes a self-similar process, because it is invariant with respect to a change of both $v$ and $u$ by the same scale factor $k$. In fact, when $v\rightarrow kv$ and $u\rightarrow ku$, $x=\int v\,du\rightarrow k^2x$ and $\langle|dh_p|^2\rangle=2\,dx\rightarrow k^2\langle|dh_p|^2\rangle$, i.e., $h_p\rightarrow kh_p$.

In order to apply these results to the analysis of BJ scaling properties, we need to clarify how Barkhausen jumps can

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be detected for a self-similar process like $v(u)$. In fact, deciding whether the DW is jumping ($v>0$) or not ($v\cong 0$) depends on one's ability to resolve fine $v(u)$ details. This can be dealt with through the introducion of the resolution coefficient $r\ll 1$ in $u$ and $v$ estimates, i.e., by assuming that we are able to measure $u$ and $v$ in units $0, r, 2r,...,$ only, so that $u\cong 0$ or $v\cong 0$ whenever $u<r$ or $v<r$. Given the resolution $r$, the mean BJ duration $\langle\Delta u\rangle$ is proportional to the probability that $v>r$, estimated from Eq. (3). With $r\ll 1$ and $c\ll 1$, we obtain

$$\langle\Delta u\rangle\propto \text{Prob}(v>r)\simeq 1-\int_0^r dv\, P_u(v)\simeq 1-r^c. \quad (4)$$

Let us now consider the distribution $P(\Delta u;r)$ of BJ durations. Given the self-similar nature of the process, $P(\Delta u;r)$ is a function of $\Delta u/r$ only, of the form $P(\Delta u/r)\sim(\Delta u/r)^{-\gamma}$. The resolution $r$ allows us to detect jumps of minimum duration $\Delta u\sim r$. On the other hand, the characteristic relaxation time of Eq. (1), equal to unity, forbids jump durations $\Delta u\gg 1$. This means that

$$\langle\Delta u\rangle\propto\int_r^1 d(\Delta u)\ \Delta u\left(\frac{\Delta u}{r}\right)^{-\gamma}\propto 1-r^{2-\gamma}. \quad (5)$$

By comparing Eqs. (4) and (5), we obtain $\gamma=2-c$.

Similar considerations can be made for the distribution $P(\Delta x;r)$ of BJ sizes. The first point is to redefine Eq. (3) when we consider the probability of finding a given $v$ value at a random position $x$ rather than at a random time $u$. It is easily checked that this introduces an extra $v$ factor in Eq. (3), which thus becomes $P_x(v)=v^c\exp(-v)/\Gamma(c+1)$. The mean BJ size $\langle\Delta x\rangle$ can be estimated from this expression in the same way $\langle\Delta u\rangle$ was estimated from Eq. (3):

$$\langle\Delta x\rangle\propto 1-\int_0^r dv\ P_x(v)\simeq 1-r^{c+1}. \quad (6)$$

Since $x$ scales like $\int v\, du$, the distribution $P(\Delta x;r)$ of BJ sizes is a function of $\Delta x/r^2$ only, of the form $P(\Delta x/r^2)\sim(\Delta x/r^2)^{-\beta}$. Under the resolution $r$, the minimum detectable jump size is $\Delta x\sim r^2$, and the cutoff at $\Delta u=1$ and $v=1$ forbids jump sizes $\Delta x\gg 1$. This means that

$$\langle\Delta x\rangle\propto\int_{r^2}^1 d\ (\Delta x)\ \Delta x\left(\frac{\Delta x}{r^2}\right)^{-\beta}\propto 1-r^{4-2\beta}. \quad (7)$$

By comparing Eqs. (6) and (7), we obtain $\beta=3/2-c/2$. Finally, it is worth remarking that Eq. (3) also has a scaling structure, with a scaling exponent, say $\alpha$, equal to $1-c$. In conclusion, we have the following result:

| | | |
|---|---|---|
| DW velocity: | $P(v)\sim v^{-\alpha}$, | $\alpha=1-c$; |
| BJ size: | $P(\Delta x)\sim(\Delta x)^{-\beta}$, | $\beta=3/2-c/2$; |
| BJ duration: | $P(\Delta u)\sim(\Delta u)^{-\gamma}$, | $\gamma=2-c$. |

## III. EXPERIMENTAL RESULTS

The BE experiments were performed on polycrystalline Si–Fe alloys. Single strips (length 20 cm, width 1 cm, and thickness 0.18 mm) of 1.8 wt % Si–Fe (electrical conductivity $\sigma=2.76\times 10^6\ \Omega^{-1}\ \text{m}^{-1}$) were placed in a solenoid and

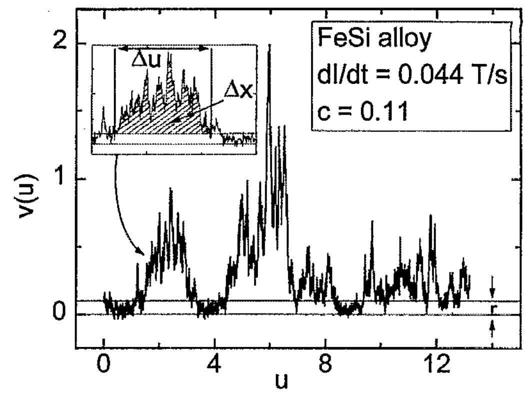


FIG. 1. Time behavior of BE flux rate $\dot{\Phi}$. Dimensionless quantities $u=t/\sigma GS\mu$ and $v=(\sigma G/AS\mu)\dot{\Phi}$ are shown. $r=0.1$ is the resolution parameter used in the determination of BJ distributions. Definition of BJ duration $\Delta u$ and BJ size $\Delta x$ is shown in the inset.

magnetized by a triangular primary current waveform of variable frequency. A flux-closing NiFe yoke was used to reduce the reluctance of the magnetic circuit. The specimen was placed in a double mu-metal box and all measurements were performed in a shielded room in order to prevent electromagnetic disturbances. The BE signal was detected by a narrow 50 turn coil placed in the middle of the strip. The noise analysis was restricted to a magnetization interval of 0.3 T around the central part of the saturation hysteresis loop. In this region, the differential permeability $\mu=dB/dH_a$ is fairly constant, and the application of a constant external field rate $\dot{H}_a$ gives a stationary BE process associated with well-defined values of $\mu$ ($\mu/\mu_0=14\,300$ in the present experiments) and of the magnetization rate $\dot{I}=\mu\dot{H}_a$ (for a more complete description of the experimental setup see Ref. 5).

According to the theory,[5] the dimensionless quantities $u$, $v$, and $x$ appearing in Eqs. (1)–(7) are defined as $u=t/\tau$, $v=(\sigma G/AS\mu)\dot{\Phi}$, $x=\int v\, du$, where $t$ is the time, $\dot{\Phi}$ is the induced flux rate per coil turn, $\tau=\sigma GS\mu$ is the time constant controlling the decay of magnetostatic fields, $S$ is the specimen cross-sectional area ($S=1.8\times 10^{-6}\ \text{m}^2$), $G=0.1357$, and $A$ is a microstructural parameter measuring the strength of local pinning interactions. The parameter $c=\langle v\rangle=(\sigma G/AS\mu)S\dot{I}$ is proportional to the average magnetization rate $\dot{I}$. Therefore, varying $c$ values were simply obtained by controlling the applied field rate $\dot{H}_a=\dot{I}/\mu$. The value of $A$ was determined through measurements of the BE power spectrum.[5] For the present material $A=8\times 10^6\ \text{A}^2\ \text{m}^{-2}\ \text{Wb}^{-1}$, which implies $c=2.6\dot{I}$.

Figure 1 shows the typical measured time behavior of $v(u)$. As discussed in the previous section, given a threshold $r$, the BJ duration $\Delta u$ is defined as the time interval between the two successive points for which the signal $[v(u)>r]$ crosses the threshold. Correspondingly, the BJ size $\Delta x$ is the area of the signal between the two points. The log–log histograms of the relative frequency of occurrence of different $v$, $\Delta u$, and $\Delta x$ values ($c=0.11$) are shown in Fig. 2. $P(\Delta x)$ and $P(\Delta u)$ exhibited a similar well-defined slope in all cases, which permitted a reliable determination of the exponents $\beta$ and $\gamma$ (broken lines). This was not the case for

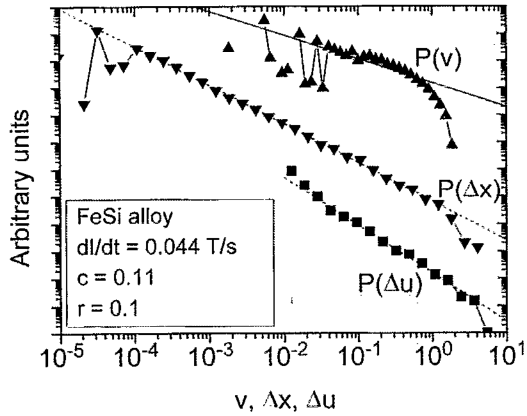


FIG. 2. Log–log plot of $v$ distribution $P_u(v)$, BJ duration distribution $P(\Delta u)$, and size distribution $P(\Delta x)$. Broken lines are best fit lines, giving exponents $\beta$ and $\gamma$. The continuous line of slope $c-1$ is shown for comparison.

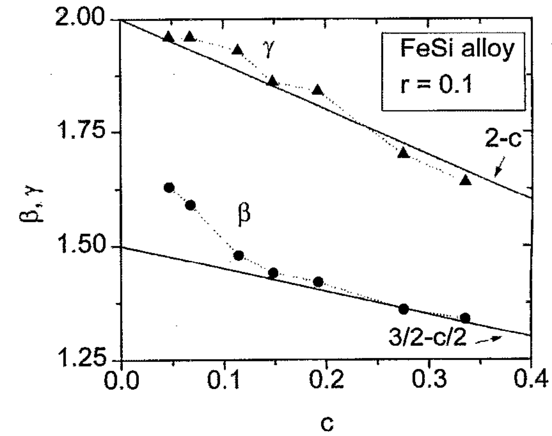


FIG. 3. Measured behavior of $\beta$ and $\gamma$ vs $c=2.6\dot{I}$. Continuous lines are theoretical predictions. No adjustable parameter is involved in the comparison.

$P_u(v)$, where the presence of large fluctuations in the distribution at low $v$ and the dominant role of the exponential cutoff at large $v$ made the determination of $\alpha$ quite unreliable. The continuous line in the figure shows that these data are, at least, consistent with the expected law $P_u(v) \sim v^{c-1}$.

The symbols in Fig. 3 show the behavior of $\beta$ and $\gamma$ vs $c$ (i.e., vs $\dot{I}$) obtained from this analysis. The continuous lines are the theoretical predictions previously discussed. Both the absolute value and the $c$ dependence of the exponents are well described by the theory. It is worth remarking that this comparison does not involve any adjustable parameter.

## IV. DISCUSSION AND CONCLUSIONS

BE scaling properties have recently been the subject of several investigations.[3,4,8,9] Interesting results were obtained[8] by applying the methods of fractal geometry to the analysis of the BE signal. These authors find that BE behaves like a self-affine process describable in terms of fractional Brownian motion processes, with fractal dimensions in the range 1.5–1.7 (the standard random walk process corresponds to the value 1.5). The analysis presented in our paper is based on the assumption that the local pinning field experienced by the DW can indeed be described as a space random-walk process. This is reflected by the fact that the distribution of BJ sizes in the limit $c \to 0$ has a scaling exponent of 1.5. In fact, in this limit the BJ sizes correspond to the segments obtained by cutting the $H_p(x)$ function with a line of constant height $H_a$ (the time variation of $H_a$ in a BJ can be neglected if $c$ is small), and it is known[10] that this gives a distribution of segment lengths with exponent 1.5.

The results obtained in Ref. 8 suggest a generalization of the present approach, where, in the equation $v \propto h_a(u) - x - h_p(x)$, $h_p(x)$ is a fractional Brownian motion process, spanning a continuous range of fractal dimensions. Future work will be devoted to the study of the scaling properties of this class of stochastic processes.


[1] H. Bittel, IEEE Trans. Magn. **MAG-5**, 359 (1969).
[2] G. Montalenti, Z. Angew. Phys. **28**, 295 (1970).
[3] R. D. McMichael, L. J. Swartzendruber, and L. H. Bennett, J. Appl. Phys. **73**, 5848 (1993).
[4] P. J. Cote and L. V. Meisel, Phys. Rev. Lett. **67**, 1334 (1991); L. V. Meisel and P. J. Cote, Phys. Rev. B **46**, 10 882 (1992).
[5] B. Alessandro, C. Beatrice, G. Bertotti, and A. Montorsi, J. Appl. Phys. **68**, 2901, 2908 (1990).
[6] G. Bertotti, in *Models of Hysteresis, Pitman Research Notes in Mathematics*, edited by A. Visintin, (Longman, London, 1993), pp. 1–9.
[7] R. Vergne, J. C. Cotillard, and J. L. Porteseil, Rev. Phys. Appl. **16**, 449 (1981).
[8] O. Geoffroy and J. L. Porteseil, J. Magn. Magn. Mat. **97**, 198, 205 (1991).
[9] H. Yamazaki, Y. Iwamoto, and H. Maruyama, J. Phys. Paris **49-C8**, 1929 (1988).
[10] M. Schroeder, *Fractals Power Laws and Correlation* (Freeman, New York, 1991), p. 153.