

# Random Field Ising Model

## Spin Models and Random Energy Landscapes

We tried to show in the introduction the intrinsic difficulties that one encounters when trying to use the micromagnetic approach. In the use of spin models it is possible to overcome some of these difficulties.

In this kind of models an object called *spin* is associated to each region of constant magnetization, say a domain. An important choice, at this level, is the definition of the object *spin*, for example if it has to be an Ising spin with just two opposite values  $\pm 1$ , or a Heisenberg spin, with constant magnitude but having the possibility to rotate in a continuous way in the space. Then, it is necessary to define the topology one is interested into – for example if it has to be a two or three dimensional lattice, and its boundaries conditions. As a last step the Hamiltonian of the system is defined. In this definition all the complex interactions between the real magnetic domains are replaced in some way that makes the system mathematically more tractable. This substitution usually involves the use of random interactions, or random fields local to each spin, or both. In any case, the use of probability distributions permits one to reproduce realistic situations, with the tuning of the parameters of the distributions.

It should be observed that the use of this kind of models can be of great interest to understand certain general aspects of the magnetization processes, but the exact solution of these models is usually not available. The only case where an exact solution was obtained is the two dimensional Ising model, where the  $\pm 1$  spins interact with constant interactions and no random fields are present.

The standard Ising model, although interesting, will not be discussed here. The reason comes from the introduction, where we discussed how our main interest lies in the feature of the magnetization process that leads to the formation of many metastable states in the free energy. A similar behaviour is not obtained in common spin systems, while it is present in the models known as spin glasses.

## Spin Models

### Spin glasses and the Sherrington–Kirkpatrick model

Spin glasses [Mezard,Parisi,Virasoro 1987; Fisher, Hertz 1991] are defined as systems possessing the two following properties: *disorder* and *frustration*; let us see how these properties arise. By the name *spin glass* we indicate both a class of magnetic materials and the models used to analyze their behaviour; in this chapter we are more interested in models, and in particular we will discuss the Sherrington–Kirkpatrick (SK) model.

The basis element of these models is the spin, where we mean by spin a random variable  $s_i$  that can take different values. The system state at any given time is then defined unambiguously by the set  $\{s_i\}$  with  $i=1..N$ . To simplify, we can let the state variables to assume just two values:  $s_i=\pm 1$ . In this case they are termed Ising spins, because they can assume just two states that can even be indicated as state "up" ( $s_i= +1$ ) and state "down" ( $s_i= -1$ ).

To introduce the frustration, we have now to add some interaction between the spins. The interaction is described by the set of random variables  $\{J_{ij}\}$ . When  $J_{ij}>0$ , the spin  $s_i$  and the spin  $s_j$  interact ferromagnetically, and their lower energy state request them to assume the same value. If instead  $J_{ij}<0$ , the couple of spins is more stable when their values are opposite. Once the set  $\{J_{ij}\}$  has been generated in some way, it is easily seen how frustration arises. Given for example just three spins  $s_1, s_2, s_3$ , if  $J_{12}$  and  $J_{13}$  are positive while  $J_{23}$  is negative, there exists no configuration where the three spins can stay in their most stable state: at least a couple of the three is frustrated.

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The disorder instead arises because the parameters  $J_{ij}$  defining the spin–spin interactions (structural parameters) are defined at random, from a given probability distribution. This choice reflects the nature of real spin glasses. A simple physical spin glass can be obtained for example from a dilute solution of  $Mn$  in  $Cu$ , where the  $Mn$  atoms are distributed at random in the  $Cu$  matrix, and their coupling oscillates as a function of the relative distance. So the presence of disorder is relevant, and it is a good choice to reflect this fact with a probability distribution for the spin–spin interactions.

The SK model introduces some simplification onto the generic spin glass model. First, the  $\{J_{ij}\}$  variables have the same probability distribution. Second, all the spins interact with each other, so that this model is even defined as infinite dimensional. Furthermore, the  $J_{ij}$  coupling variables are symmetrical:  $J_{ij} = J_{ji}$ .

The starting point of this model is its hamiltonian:

$$H = -\frac{1}{2} \sum_{i,j=1}^N J_{ij} s_i s_j - H \sum_{i=1}^N s_i \quad [1]$$

where the sums are on all the  $N$  spins of the lattice, and the second term merely indicates the coupling of the spins with the external field  $H$  applied on the system. Very often the second term is neglected, since many studies concentrated on the properties of the system without any external driving force. We include it here because our approach is different – as it will become evident when discussing the RFIM model – and our interest lies in the hysteretic properties of these systems [Bertotti, Pasquale 1990, 1991].

Since when we discussed the property of frustration we saw that the spins cannot satisfy *all* the bonds with their neighbours, one expects that in an infinite dimensional system the frustration will reach the highest levels, and the system will be unable to find a stable state. This is an erroneous point of view however: a system like this has a huge number of stable states, where we do mean by stable that the energy is at a minimum for small changes of the system state. For example, one can find a stable state in which the energy  $E_0$  will be the lower in regards to one–spin flips: whichever spin we invert, the energy will be higher. Notice that this does not exclude the existence of lower energy states obtained by flipping two spins at the time, but the notion of *local* minimum implies a definition of small system state displacements, and the one–spin–flip is the smallest displacement possible.

The SK model does not exhibits ergodicity, in the thermodynamic limit  $N \rightarrow \infty$ . This happens because if the system state is trapped in a local minimum it will remain there forever, due to the fact that the height of the energy barriers too goes to infinity. Instead, statistical mechanics requires in its calculations to perform an average over all the minima with appropriate weights. The difference between time averages and ensemble averages is named nonergodicity. We describe en passant this property because it is most important in the relaxation effects in complex energy landscapes (we will talk about relaxations later, in §5). Nonergodicity could in fact explain the fact that complex systems show very low relaxation times, as the system temperature is lowered below the critical temperature.

### The Random Anisotropy Model

The Random Anisotropy Model (RAM) differs from the spin glass model in a fundamental way. Once one agrees that in magnetic materials the disorder can manifest itself basically in two ways – disorder in the exchange interaction and disorder in the local anisotropy – it is possible to see that spin glass models, and in particular the SK model, implement in a simple model the variation of the exchange interaction. The RAM [Alben, Becker, Chi 1978; Chudnovsky 1986, 1988; Chudnovsky, Serota 1982; Chudnovsky, Saslow, Serota

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1986; Hernando, Vazquez, Kulik, Prados 1995] structures its hamiltonian to reflect the fact that in the material the presence of short range structural order gives rise to local random anisotropy axes. Along these axes the magnetization is forced to lie, while a *constant* exchange interaction forces in a parallel direction the spins. Last, the presence of an external field along an arbitrary direction will force the spins along its direction. The resulting hamiltonian is:

$$H = -J \sum_{\langle \vec{p} \rangle} \vec{s}_i \cdot \vec{s}_j - K \sum_i \vec{n}_i \cdot \vec{s}_i - H \sum_i \vec{s}_i \quad [2]$$

Here we have not defined the spin lattice. Let us just say that in the exchange term the spins interact just to the first neighbour level. The constant  $K$  is the anisotropy constant, and the vectors  $\vec{n}_i$  define the local anisotropy at each location  $i$ . An interesting assumption used here is the randomness of these vectors, but under the constriction they be locally correlated, with correlation length  $R_a$ . This is a realistic assumption: if we take for example a non-oriented crystallized sample, we can observe that in each grain the magnetization is forced to follow a local anisotropy axis. If our description level is lower than that of a single grain (that is, if we assume that each spins represents the magnetization of an elementary volume smaller than a single grain) we must use a random anisotropy correlated on the linear dimension of the grain.

This model has proven itself apt to describe well many properties of amorphous magnets, since these are typical materials in which the local anisotropy varies in a continuous way.

### The Random Field Ising Model

The Random Field Ising Model (RFIM) [see for example Dahmen *et al.* 1993, 1996; Sethna *et al.* 1993; Behn *et al.* 1990; Belanger, Young 1991; Garrido, Marro 1989; Grinstein, Ma 1982; Hai, Li 1989; Rieger, Young 1993; Villain 1984] contains some simplifications in respect to the models described so far. The spins we

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consider are Ising-like, thus just two values  $s_i = \pm 1$  are allowed. The interaction strength is defined with constant value  $J > 0$ , thus it is ferromagnetic-like: the energetic preferred state is with parallel spins. The disorder is introduced in the so called random field: at each spin location  $i$  is present a field  $h_i^c$  extracted from a pre-defined distribution, that forces the spin  $s_i$  to lie in its direction. The term *direction* used in this context means simply that a positive random field forces a  $s_i = 1$  value, while a negative random field forces its value to  $s_i = -1$ . Let us note here that the disorder is quenched. The system is completely defined from the beginning with the  $h_i^c$  initialization, then the random fields are no more modified, and the system is let free to evolve under the action of an external field that acts along the direction of the spins, again forcing a positive spin value if  $H$  is positive, and a negative spin value if  $H$  is negative. The model is not infinite dimensional: in our study we analyzed the two and three dimensional cases, with  $N$  spins, where the spin-spin interactions are defined in a neighbourhood of 4 (2D) and 6 (3D) spins. Both the open and periodic boundary conditions have been studied. The Hamiltonian of our system is:

$$H = -\frac{J}{2} \sum_{\langle ij \rangle} s_i s_j - \sum_i s_i h_i^c - N \cdot H \cdot M \quad [3]$$

where the total system magnetization is defined as  $M = \frac{1}{N} \sum_i s_i$ . The coupling constant, as we mentioned, is

ferromagnetic like:  $J > 0$ . It can be observed the similarity of the hamiltonian with that of the SK model [Eq.1].

The first term is the interaction term that forces the spins to lie in the same direction. The second term is the disorder term, that forces each spin to lie along the direction of the local field. The last term represents the interaction of the external field with the single spins: each field is forced to lie in the direction of the external field. The first sum is restricted (as indicated with the notation  $\langle ij \rangle$ ). Its meaning is that we have to perform a sum over all the spins  $i$  on the lattice, and for each spin a second sum over its  $j$  neighbouring spins, that is over 4 or 6 spins for the two and three dimensional lattice.

The system is considered to be at zero temperature. Clearly, a  $T=0$  treatment is highly idealized, and there exist many possible algorithms to simulate the intervention of the temperature in the spin flipping, with statistical methods. But our intent was to study in detail, as we will discuss, the magnetization dynamics as a succession of instabilities triggered by the magnetization reversal in localized regions of the system, in a perfect analogy to the Barkhausen jump phenomena. And spin flipping events triggered by a temperature fluctuation would lead to highly nontrivial complications in the interpretation of the results.

Another useful way to write the hamiltonian is the following:

$$\begin{cases} H = -\frac{1}{2} \sum_i s_i h_i & [4] \\ h_i = J \sum_{\langle k \rangle_i} s_k + h_i^c + H \end{cases}$$

where by  $\langle k \rangle_i$  we indicate the spins  $k$  in neighbourhood of the spin  $i$ . The field  $h_i$  represents the internal field acting on the spin  $s_i$ . The interest of this definition can be understood when one is trying to obtain a relationship that defines the stability conditions for the system. As we mentioned, the energy landscape in the RFIM model is extremely rugged and characterized with a multi-valley structure, in analogy with what happens in real systems. Conversely, it is a model with a well defined hamiltonian, and this fact suggests that we should be able to obtain some simple relationship between its energetic properties and the structure of the energy landscape. Having written the hamiltonian in the form [Eq.4], we see that the stability condition, that gives the configuration  $\{s_i\}$  in which the energy is locally at a minimum is:

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$$s_i = \text{sgn}(h_i) \quad [5]$$

that is, every spin is collinear with its internal field. In this case the term  $-\frac{1}{2} \sum_i s_i h_i$  is obviously at the minimum. Of course, it is just a *local* minimum, since there can exist many stable configurations  $\{s_i\}$  for which [Eq.5] holds. What is important is that this equation shows that if any spin flips, the energy will be higher. Thus we have stability in front of one-flipping-spins. Nothing we can say in front of multiple spin flips. This equation gives a prescription to obtain a stable configuration, at fixed external field. But, once obtained, it is possible that reversing a certain number of spins we would obtain an instability in the form of spin avalanche that could lead to another stable state with lower energy.

From the overview of the three terms in the hamiltonian we observe that they can give rise to many metastable configurations. The energy landscape will then be very complex, as a function of the system state.

The system state is exactly defined with the  $N$  coordinates  $\{s_i\}$ , so, as previously discussed, no approximation will be necessary to define it, using the total magnetization or the domain structure. Although the disorder is quenched from the beginning, the energy landscape will change continuously as the external field is changed. We already discussed this phenomenon: slowly, the stable configurations will become unstable as the external field will transform points of energy minimum of the energy landscape in saddle points, and then in unstable states. At this point the system will find itself at the threshold of an instability: [Eq.5] will no longer hold and the spin will have to rearrange themselves to come back at a minimum configuration. Doing so the system state will change, being defined with the coordinates  $\{s_i\}$ .

### RFIM properties

#### The avalanche

Let us analyze in greater detail what happens during a spin avalanche. We start from a given stable state  $\{s_i\}$ , and we change the external field  $H$  so slowly that, in the case of instability, its value practically does not change during the time needed for the system to attain again an equilibrium state. The stability condition [Eq.5] leaves all the spins  $\{s_i\}$  unchanged, each along the direction of its internal field  $h_i$ , until one of the internal fields will cross the zero value. Let us take note of the fact that, for any finite number  $N$  of spins, *just one spin* can have exactly a zero internal field value. The internal field having crossed the zero value, the spin will be forced by [Eq.5] to flip, so to be stable again. But its inversion will change the internal field value for its 4 (or 6) neighbours, due to the interaction term: the internal field of each spin will change of a factor  $\pm 2J$ . At this point, two things can happen. If the neighbouring spins are still stable, what we observed is an avalanche composed of a single spin flip, and the interesting point here is that this phenomenon is *reversible*: changing the direction of the external field variation, the reverse process will happen, and that single spin will flip back. If instead one or more of the neighbouring spin in turn becomes unstable, the analysis must be repeated: those spins must be reversed, and the instability has the possibility to propagate through the lattice. The avalanche stops just when [Eq.5] is satisfied again by all the spins. The total number of flipped spins is called the size of the avalanche. This process is *irreversible*, since the spins from the second one on become unstable at a non null internal field  $h_i$  such that  $0 < h_i < 2J$  or  $-2J < h_i < 0$ , and when the spin is flipped we have an irreversible loss of energy.

If the process is reversed, a different sequence of spin flips will happen. If the first spin that triggers an

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avalanche makes more than one spin unstable, the question can arise about the order in which we have to flip the unstable spins. It can be verified that it does not make any difference. Being all the interactions ferromagnetic, it is impossible that an unstable spin not yet turned will be made stable by another spin flipped before it. This is in contrast with what can happen in the SK model, where instead the interactions  $J_{ij}$  have random sign. In this case it happens that unstable spins be made stable before they are chosen to flip. As a consequence, the flipping order must be chosen carefully in the simulations, usually assigning randomly a relaxation time for each spin, and flipping them in time order.

Similar systems, in which on a given lattice some "event" occurs, triggered by some external action, and propagates through the lattice until finally dying out, have been studied for years. The class they generally belong to is the class of cellular automata [Wolfram 1994], and one of the best known is the so-called "Game of Life", a 2D cellular automaton with a small set of transition rules for the state variables, which can assume just two values. Recent wave of interest on self-organized criticality (SOC, see §4) lead to some interesting conclusion on these automata: indeed the "Life" automaton has been demonstrated to exhibit the SOC properties [Bak,Chen,Creutz 1989; Sales 1993].

The question then naturally arises, is SOC behaviour present even in the RFIM model? We didn't yet have time to address this problem, but we consider it an interesting field of study.

### Partial ordering, and the no passing rule

An important relationship that can be easily defined in the Ising systems is known as partial ordering. Given two configurations  $\{s_i\}$  and  $\{r_i\}$ , we have partial ordering  $\{s_i\} \geq \{r_i\}$  if  $s_i \geq r_i \forall i$ . As an example, in a 2D case:



$\{s_i\}$



$\{r_i\}$

Fig.1 Partial ordering of states, case two dimensional. Black locations are  $s_i=1$ , white locations are  $s_i=-1$ . In the case shown, the states are partially ordered and  $\{s_i\} > \{r_i\}$

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Please observe that, contrarily to what can appear, this is not a simple relationship regarding the total magnetization. If there exists a partial ordering relationship between  $\{s_i\}$  and  $\{r_i\}$ , and  $\{s_i\} \geq \{r_i\}$ , it is also true that  $M_{s \geq} M_r$ . But there can exist two states such that  $M_{s \geq} M_r$  which are not partially ordered. This relationship is a *local ordering* relationship.

The importance for the RFIM model of the partial ordering relationship is that *partial ordering is conserved by the evolution dynamics*: a law known as "no passing rule" [Middleton 1992].

Given the same system – that is, being defined the same random internal fields – we consider two possible states  $\{s_i(t)\}$  and  $\{r_i(t)\}$ , dependent on time. Let us suppose that the first state evolves under the action of the external field  $H_s(t)$ , while on the second one is applied the field  $H_r(t)$ , with  $H_s(t) \geq H_r(t)$ . If at a given time  $t_0$  these two states are partially ordered:  $\{s_i(t_0)\} \geq \{r_i(t_0)\}$ , then it follows that  $\{s_i(t)\} \geq \{r_i(t)\} \forall t > t_0$ .

In fact, if a time  $t'$  exists at which a spin  $j$  breaks the partial ordering relationship:  $r_j(t') > s_j(t')$ , this implies, from [Eq.5], that the internal fields too are such that  $h_j^{(r')} > h_j^{(s')}$ . That is, it should be:

$$J \sum_{\langle k \rangle_j} r_k + h_j^c + H_r > J \sum_{\langle k \rangle_j} s_k + h_j^c + H_s \quad [6]$$

and this cannot be true. In fact, we have here the same local random field  $h_j^c$ . In addition  $H_s > H_r$ , and, being the spin  $j$  that we are considering the first one that violate the partial ordering relationship, every spin in neighbourhood of  $j$  still satisfies it:  $s_k \geq r_k$  for every  $\langle k \rangle_j$ . So it follows that  $h_j^{(s')} \geq h_j^{(r')}$ , and  $s_j(t') \geq r_j(t')$ .

### Wiping-out

The wiping-out property (or return point memory) is another property typical of this model. Its importance is great, in particular when comparing this with other magnetic models, like the Preisach model or the ABBM model that will be introduced later, and in the comparison with real materials. It is a property that enables the system, when subjected to a non monotonous field history, to store the memory of the state at every field reversal point, so that when the field will reach again that value for the first time, the system state will be exactly the same.



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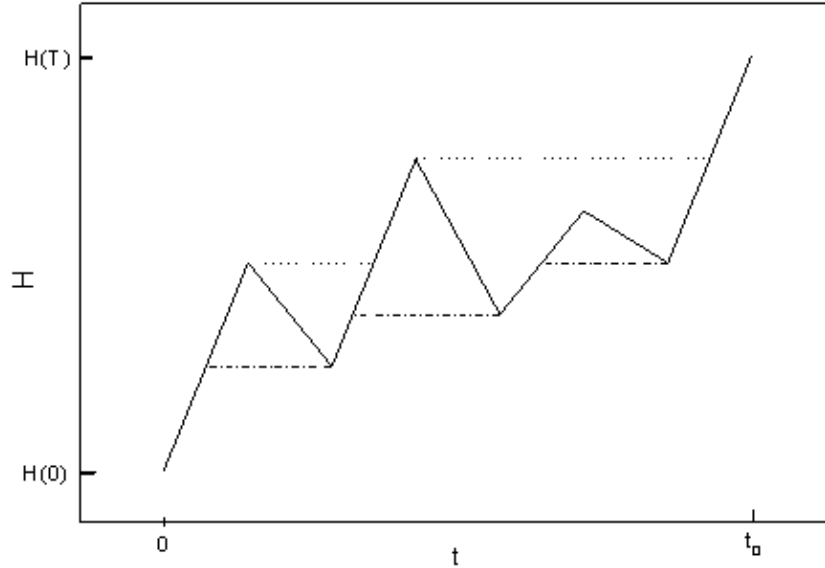


Fig.3 The three fields used in the wiping-out demonstration:  $H(t)$  (solid line),  $H_{min}(t)$  (dot-dashed line), and  $H_{max}(t)$  (dotted line)

Corresponding to the three field histories here defined, we will consider three states:  $\{s_i\}_{min}$ ,  $\{s_i\}$ ,  $\{s_i\}_{max}$ , evolving respectively under  $H_{min}(t)$ ,  $H(t)$ , and  $H_{max}(t)$ , and such that  $\{s_i(0)\}_{min} = \{s_i(0)\} = \{s_i(0)\}_{max}$ . Now, being  $H_{min}(t) \leq H(t) \leq H_{max}(t)$ , the partial ordering rule gives  $\{s_i(t)\}_{min} \leq \{s_i(t)\} \leq \{s_i(t)\}_{max}$ , and, from the adiabaticity of the system:  $\{s_i(t_0)\}_{min} = \{s_i(t_0)\}_{max}$ . That is,  $\{s_i(t_0)\}$  is independent from the field history, but just depends on the maximum field value  $H(t_0)$ .

### The simulations

We wrote a computer program able to simulate the behaviour of the RFIM model, to study certain aspects of its behaviour. We were interested in particular in its study under the action of a slowly varying external field. So we analyzed the hysteretic behaviour, the statistical properties of the avalanche size distribution, and the energy landscape present during the magnetization process. The demagnetized state properties have been studied too. The key point of these studies have been the role of disorder in the propagation of the instabilities in the system.

### Simulation parameters

#### Number of spins

In the case of a computer simulation the number  $N$  of spins considered cannot be too large, or the time required could grow beyond acceptable limits. In the case of  $N=20000$ , the time needed to perform a hysteresis loop is of the order of  $1000 \text{ sec}$  on a Digital ALPHA workstation, and grows more than  $O(N^2)$ . From the other point of view,  $N$  values too small would cause a great loss of details, and the boundary

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conditions could superimpose their effects on the behaviours searched.

The  $N$  value we used was never smaller than 900, and to acquire greater detail we arrived up to values of  $N=40000$ . A valuable method in these cases is even to collect the results from many simulations performed over systems statistically equivalent. Due to the non-linear growth of the computational time as a function of  $N$ , this method allows to spend lesser time and to greatly reduce the noise. Of course, however, the boundary condition effects do not reduce in this way, so in any case  $N$  must not be too small.

### Order/Disorder relationship

We showed in the hamiltonian

$H$  [Eq.3] that there are two terms that describe the order and the disorder present in the system: the interaction term  $-\frac{J}{2} \sum_{\langle ij \rangle} \varepsilon_i \varepsilon_j$  has

a strength  $J$  and has the effect of keeping the spins aligned in the same direction, although acting just at the nearest neighbour level (an extension of this model could allow longer range interactions). And the random field term  $-\sum_i \varepsilon_i h_i^c$  that tries to align every spin

along the direction of a local random field. In our simulations this *frozen disorder* is generated at the beginning of the simulation using a gaussian distribution:

$$P(h_i^c) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2} \left(\frac{h_i^c}{\sigma}\right)^2\right) \quad [8]$$

with null average value and variance  $\sigma$ . The greater the value of  $\sigma$ , the greater the dispersion of the  $h_i^c$  values, and the disorder on the lattice. In the following we will use the half-width  $R$  of the gaussian distribution, related to  $\sigma$  by the relationship

$$R = \sqrt{2 \ln 2} \cdot \sigma \quad [9]$$

The random extraction of the  $h_i^c$  fields can be considered a good approximation. A more realistic approximation, following the case discussed in the RAM, can be that of randomly extract these fields, but imposing a correlation length. In this way we could realistically model the local disorder.

Every order – disorder relationship is clearly connected to the relative importance of the terms  $J$  and  $R$ . Sethna *et al.* observed how the tuning of the ratio  $R/J$  allows to pass from a region ( $R \gg J$ ) where small avalanches are present, involving small fractions of the system and whose size become negligible in the limit  $N \rightarrow \infty$ , to another region ( $R \ll J$ ) in which the instability involves a finite part of the system, thus not going to zero in the thermodynamic limit. The first region is dominated by the disorder, because on the average the local field  $h_i^c$  is greater than  $J$ : so the spins can flip just when the external field is  $H \approx h_i^c$ . In the limit of  $R/J \rightarrow \infty$  just one spin at a time is able to meet this condition. In the second region the ordering effect due to the spin–spin interactions allows the system to become globally unstable as a small number of spins becomes unstable.

Along the hysteresis loop, we observe that in the first regime the loop has a small width and the magnetization varies continuously. In the second regime instead we have observed small avalanches (precursors), followed by a large avalanche around the coercive field (named infinite avalanche), followed by other small avalanches (successors) that close the loop to saturation. The derivative of the function  $M(H)$  in this case has a singularity at  $H_c$ .

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### Boundary conditions

Two types of boundary conditions are possible. In the first case (*open boundary condition*) the spins on the boundary have a smaller number of neighbours (2 or 3). This fact has two effects: these spins are less bounded to the lattice, the absolute value of the total interaction strength being no more than  $4J$  or  $6J$ , so it is more difficult to make them flip; moreover, since beyond them there is nothing more, these spins create a barrier, and the avalanches created near the boundaries have fewer directions along which they are able to expand.

In the second case (*periodic boundary condition*) we annihilate the effect of the boundary by closing the lattice on itself, like using a toroidal topology; the conditions on the boundary spins for the 2D case are:

$$\begin{aligned} s_{i,N+1} &= s_{i,1} & \forall i & \\ s_{N+1,j} &= s_{1,j} & \forall j & \end{aligned} \quad [10]$$

where in this syntax we passed from the syntax  $s_i$  ( $i$  spin index) to the syntax  $s_{i,j}$  ( $\{i,j\}$  spin coordinates). Using the periodic boundary conditions, no barrier at all is put along the propagation of the avalanche. Of course this has no consequences at all on the spurious effects caused by finite dimension of the system.

### The hysteresis loop

The analysis of the hysteretic behaviour can lead to some important conclusions about the properties of the model. As we already mentioned, the erasing of the memory of the previous system states at a reversal point has a clear counterpart in the hysteresis loop, where the exact closure of the minor loops can be verified. The equality of the states at the reversal point has been verified using an overlapping function:

$$q = \frac{1}{2N} \sum_{i=1}^N (1 - s_i^{(1)} s_i^{(2)}) \quad [11]$$

If this function is calculated, for example, at point  $D$  in Fig.2,  $\{s_i^{(1)}\}$  is the state when the field grows from  $H_A$  to  $H_B$ , while  $\{s_i^{(2)}\}$  is the state after the sequence  $H_C-H_D-H_E-H_D-H_B$ . The overlap function should return exactly zero in the case of perfect overlap, and this has been verified in all the simulations we performed.

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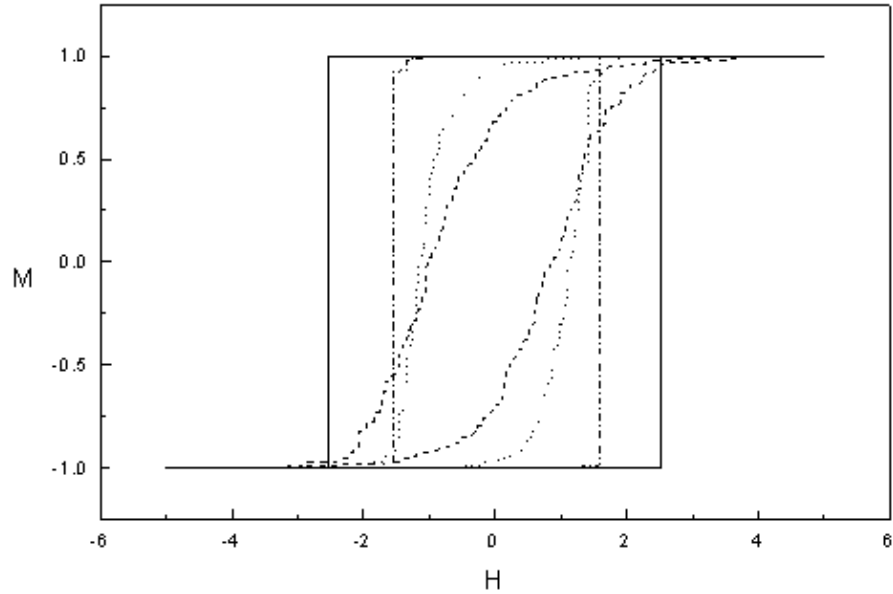


Fig.4 RFIM hysteresis loops, at various degrees of order/disorder.  $N=900$  spins,  $R=0.5$  (straight line);  $R=1$  (dot-dashed line);  $R=2$  (dotted line);  $R=3$  (dashed line)

We mentioned that the ratio  $R/J$ , changing the disorder/order relationship, has deep influences in the hysteresis loop shape. Some example can be viewed in Fig.4 . An interesting case is the  $R=0$  case, where no disorder at all is present, being  $h_i^c=0 \forall i$  . This brings us back to the Ising model case (at  $T=0$ , of course), where we have exactly a bistable system. In this situation of perfect order all the spins are exactly equivalent, so that the internal field of every spin will cross the zero value at the same external field  $H = \pm 4J$ , with the sign varying depending on the direction of the field variation.

Now, let us increase the disorder, just to a small value: we take  $R \ll J$  but still different from zero. In any case now the internal field will be zero just for one spin at the time, since we extracted the  $h_i^c$  value from a continuous distribution. We can suppose to increase  $H$  from the negative saturated state. The first spin  $s_i$  to be made unstable will be the one that has the largest  $h_i^c$  value, since  $H = -4J - h_i^c$ . We are then in the queue of the  $h_i^c$  distribution, and few spins will have  $h_j^c$  values close to  $h_i^c$ . Then the first avalanches will be small. But as we come nearer to  $H_c$  more and more spins will have similar  $h_i^c$ , since  $R$  is very small. At  $H_c$  then, a single spin flip will trigger the infinite avalanche. The remaining spins not yet flipped will be the ones with  $h_i^c$  negative and very large; again, we are now in the queue of  $P(h_i^c)$ , and this spins will turn in small clusters.

Increasing more the  $R$  value,  $P(h_i^c)$  will be wider, and the trend described for the spins to turn independently will increase. There will be a greater prevalence of small Barkhausen jumps, and the loop will be thinner, until we will arrive to a loop showing paramagnetic behaviour: every spin now turns independently (then in a reversible fashion, as we discussed) and  $H_c=0$ .

### Random Field Ising Model

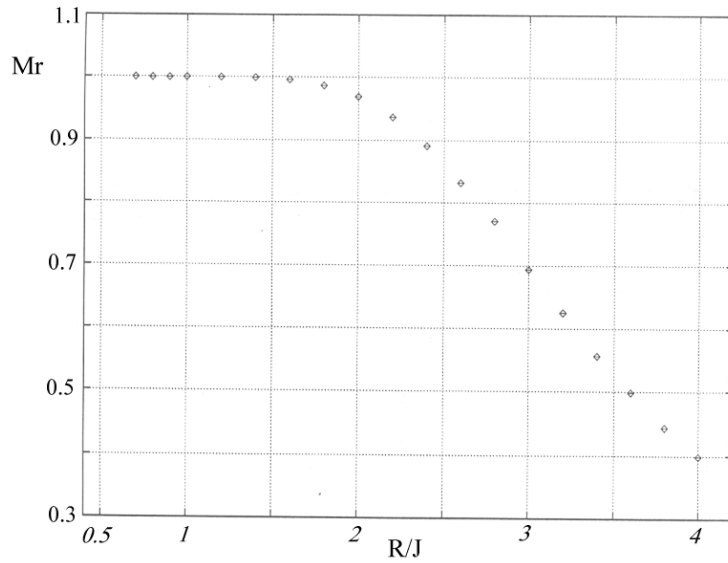


Fig.5 Remanence dependence on the  $R/J$  ratio

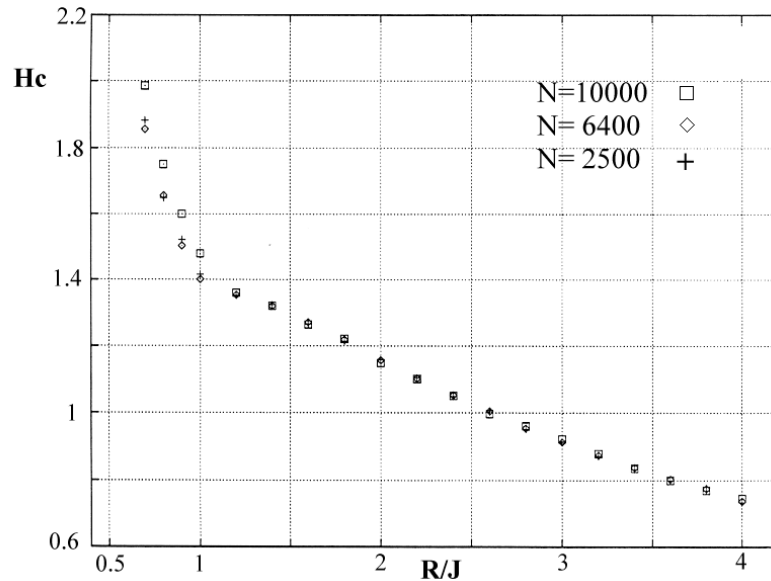


Fig.6 Coercive field dependence on the  $R/J$  ratio. Simulations with  $N=2500, 6400, 10000$  spins

We studied this behaviour both visually on the loops, and observing the remanence (Fig.5 ) and the coercive field (Fig.6 ) as a function of  $R/J$ . The remanence  $M(H=0) \equiv M_0$  remains at the value of  $M_0=1$  for low  $R/J$  values, until  $R/J \approx 1.5$ . In this region the loops behave as the so-called *squared* loops, and the system leaves the saturated state after an inversion of the sign of  $H$ . After a "knee" in the curve for  $1.5 < R/J < 2$ , the remanence starts decreasing with a linear behaviour.

The coercive field behaviour shows a finite-size effect for low disorder values. Being the system size finite, the system is able to realize more easily the conditions to assume the squared shape. In fact, this effect appeared to decrease as we increased the system size from  $N=2500$  to  $N=6400$ , up to  $N=10000$ . Beyond the  $R/J=1.2$  value, the coercive field too shows a linear decrease with the disorder ratio.

## Random Field Ising Model

### Avalanche size distribution

Before discussing the avalanche distributions, let us spend some word regarding the possible influence of the algorithms used on their evolution.

The evolution of the instability, as the avalanche proceeds, is completely deterministic. Contrarily to the SK model, the algorithm that selects the next spin to turn among the unstable spins does not affect the overall avalanche properties. In fact, as a spin that flips makes more than one spin unstable we need to decide in which order to invert the unstable spins. Whatever decision we take we won't influence the system, and this is in opposition with, for example, the SK model. The reason comes from two facts: the coupling of all the spins is ferromagnetic, and we are in adiabatic conditions. As the avalanche proceeds it is impossible that an unstable spin  $s_{i0}$  still waiting to be flipped be made stable from the flipping of other spins  $s_i$ , possibly in its neighbourhood: any spin flip of this kind will make the spin  $s_{i0}$  even more unstable. Moreover, being  $\tilde{H} \rightarrow 0$ , in any case the external field won't be different from the beginning of the avalanche. The conclusion is that at the end it will have to be flipped in any case.

So we conclude that the only factor deciding the order in which the spins have to be inverted is the set  $\{h_i^c\}$ . And, being this disorder quenched for the system, all the successive loops made between the same maximum external field values will be composed by *exactly the same sequence of avalanches, composed by the same spins*. This factor too is not verified in the SK model, where a previous decision must be taken at the algorithmic level, regarding the flipping order. In any case, some kind of random decision must be taken, and the successive loops in that model are similar just at the statistical level.

### Avalanche topology

Some observation can be done on the topology of the spin avalanche. As we mentioned, this event is analogous to a Barkhausen jump in real materials. In the RFIM case, at the end of an avalanche, a certain number of spins will be in the opposite state. Since the system is defined on a lattice, we have the possibility to observe the two or three dimensional cluster that constitutes the avalanche. This cluster must be *connected*, because the propagation of the instability takes place via the interaction term  $-\frac{J}{2} \sum_{\langle i,j \rangle} s_i s_j$  in

**H.** The dimension  $n$  of the avalanche gives an exact measure of the height in magnetization  $\Delta M$  of the Barkhausen jump, and will give an approximate measure of the energy loss. But the shape in the lattice of the avalanche is important. Let us suppose an avalanche starts at a site  $i$ . A spin  $j$  neighbouring to  $i$  can exist whose internal field variation  $\pm 2J$  is not sufficient to let it flip. But if the avalanche evolution let other spins neighbouring to  $s_j$  to change their state, the flipping probability of  $s_j$  increases. The conclusion is that the probability  $P(n)$  to have an avalanche of size  $n$  will depend too on the possible shapes that an avalanche of this size can assume. A simple example is sketched in Fig.7 .

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Fig.7 The only two possible topologically different avalanche configurations with  $s=3$ . Black squares indicate the flipped spins, the lines indicate the bonds between them

Here are shown the only two possible different avalanche configurations with  $s=3$ . A direct calculation shows that the first configuration has a relative probability to occur  $p=1/3$ , while for the second one  $p=2/3$ . But when one observes how the spins are forced to flip, i.e. with the increment in their internal field due to the interaction, realizes that the non-flipped spin in the second configuration (indicated with a dot only) received an internal field variation  $\pm 4J$ , not just  $\pm 2J$ . This implies a smaller probability for the second configuration, in respect to what can be topologically calculated (we can consider it more unstable), because the fourth spin became more unstable than the rest of the spins neighbouring to the cluster. We didn't yet performed any detailed calculation regarding the effect of the avalanche shape on the distribution of the sizes of the avalanches, but it seems clear that two factors should be taken into account: the cluster compactness (area to perimeter ratio) and the cluster convexity. In fact, the more compact the cluster, the fewer the number of spins whose internal field changes by the amount  $\pm 2J$ . And the more convex is the cluster, the fewer are the spins whose internal field increases by  $\pm 2J$ .  $n$ , where  $1 < n < 4$ . A possible way to measure this couple of factors could be via a list of four numbers,  $(n_1, n_2, n_3, n_4)$ , where to record the number of neighbours of the cluster (that is, the freedom paths in the game of *Go* [Aroutcheff 1986]): in  $n_1$  the number of one-connected neighbours, that is the number of spins neighbouring to just one of the spins belonging to the avalanche. These spins received just one contribution  $\pm 2J$  each to their internal field. Then in  $n_2$  the two-connected neighbours, in  $n_3$  the three-connected neighbours, up to the four-connected neighbours, in  $n_4$ , the rare cases of spins that can be viewed as "holes" in the avalanche, and that notwithstanding the fact that all their neighbours flipped, and that their internal field changed of a term  $\pm 8J$ , didn't flip. As an example, an avalanche of four flipped spins with linear shape would have  $(10,0,0,0)$ , while if the shape (always with 4 spins) would be an L-shape, then the list would be  $(8,1,0,0)$ . Some kind of *instability measure* of the cluster could be obtained by  $n_i / (\max(i) / n_{i \neq 0}, i \in (1..4))$ .

### The distribution

As we mentioned, the sequence of avalanches as the system gets magnetized is always the same, given the same starting condition. This is due to the dependence of the instability condition on the set  $\{h_i^c\}$ , that is given from the beginning. But it is not possible – although it could be in principle – to study this sequence abstractly, given just the set of local random fields. So we tackled this problem studying the distribution  $D(n)$  of the size  $n$  of the avalanches, that is the relative frequency with which the system shows an avalanche composed of  $n$  spins.

To override the problem we mentioned, regarding the impossibility to simulate an infinite lattice, we limited ourselves to a number of spins  $N \leq 40000$ , and obtained the distribution  $D(n)$  from a statistical ensemble obtained from the repetition of the same magnetic history for a given number of times ( $r=1000$ ) over systems

## Random Field Ising Model

statistically equivalent (same values of  $R$  and  $J$ ), but in which new random  $\{h_i^c\}$  values were extracted before each new simulation. This procedure allowed us to reduce the noise. Another useful method of noise reduction was the collection in channels whose width was the logarithm of  $n$ , so to be able to eliminate non influent fluctuations.

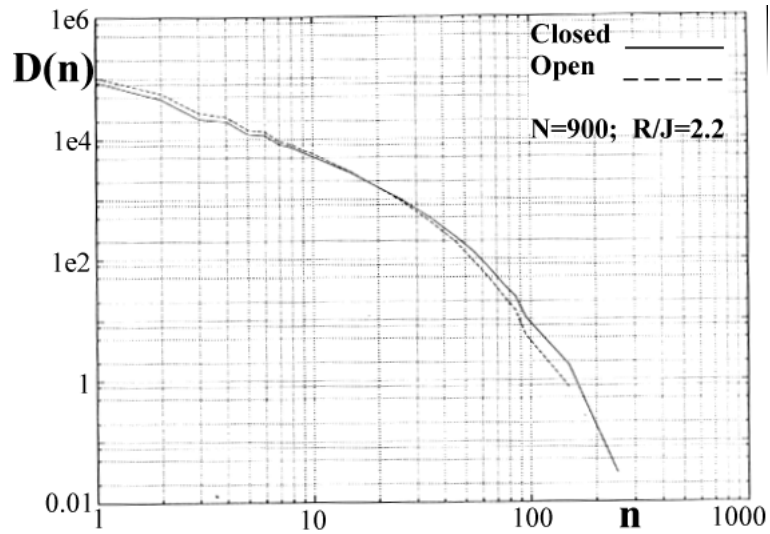


Fig.8 Influence on  $D(n)$  of the boundary conditions,  $R=2.2$ ,  $N=900$

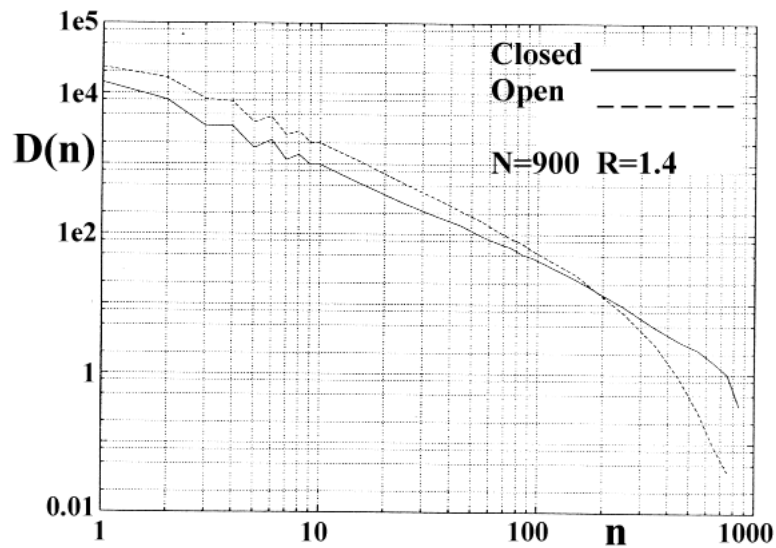


Fig.9 Influence on  $D(n)$  of the boundary conditions,  $R=1.4$ ,  $N=900$

## Random Field Ising Model

A preliminary study regarded the influence of the boundary conditions (b.c.), so the same investigation was performed with periodic b.c. and with open b.c. In Fig.8 and Fig.9 this effect can be observed, in conditions of high and low disorder. The b.c. effect is stronger when the disorder is lower. In fact, in the case of low disorder a single spin flip can trigger a big instability, but big instabilities are hampered in the case of open b.c., which decrease the number of directions along which an avalanche can safely evolve. This also explains the fact that  $D(n)$  decreases more quickly in the case of open b.c. at high  $n$  values. After this test, periodic boundary conditions were used in all subsequent simulations.

A question that now deserved an answer regarded the possible existence, in the bidimensional model, of a value for the ratio  $R/J$  sharply discriminating between order and disorder. Sethna showed that in the three dimensional case the transition between the order and regions is clearly defined, and appears at  $R_c = 2.23 J$ . This kind of study is usually performed with analytical methods. But some indication can also be obtained when one recalls that at the critical point the correlation length should go to infinity, as phenomena on all the length scales can appear. In our case, this means that we should observe avalanches of any size (let us recall that the correlation length can be connected to the square root of the number of the spins of the avalanche), and that the law expressed for their distribution should be exactly a power law.

With the study of these distributions, it is possible to discriminate the two different regions we talked about.

In the low disorder limit ( $R \ll J$ ) the distribution shows a peak corresponding to the average size of the infinite avalanche, and smaller peaks corresponding to the precursor and successor avalanches.

As the disorder increases, the distribution form tends to the limit of a power law:

$$D(n) \approx n^{-\tau} \quad [12]$$

expressing the criticality of the system as an absence of typical length scales.

When  $R > R_c$ , an exponential cutoff appears in the power law

$$D(n) \approx n^{-\tau} \exp(-n/n_0) \quad [13]$$

expressing the presence of a typical length scale  $\sqrt{n_0}$ , whose cause is given by the impossibility for the system to reverse a non determinate number of spins during the avalanche: the  $h_i^c$  values have a too wide distribution now. Fig.10 , Fig.11 show this mutating behaviour for the  $N=900$  and  $N=4900$  cases.

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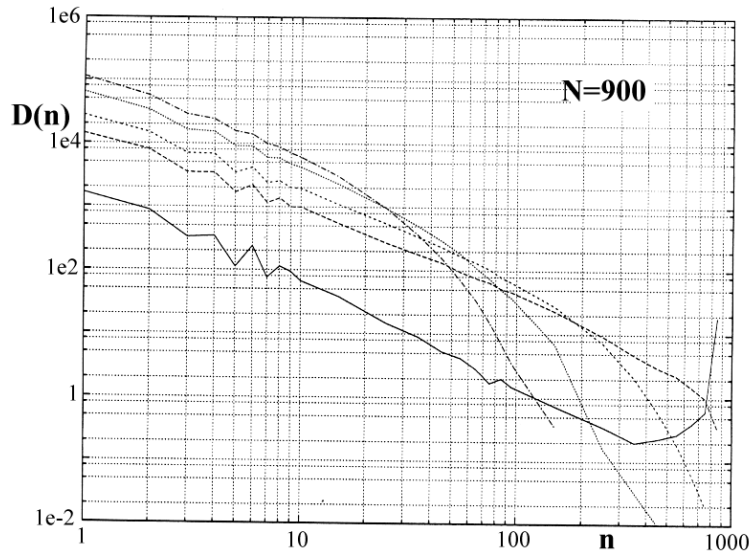


Fig.10 Distribution  $D(n)$  for  $R/J=2.4, 2.0, 1.6, 1.4, 1.0$ ; open b.c.;  $N=900$

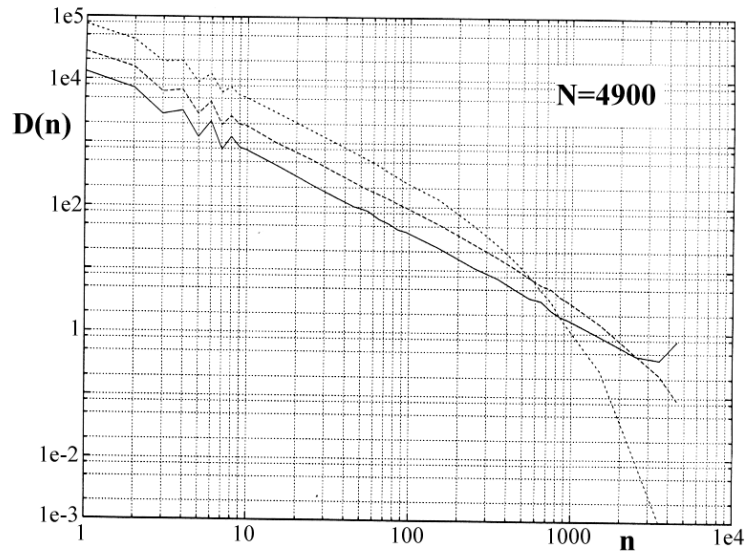


Fig.11 Distribution  $D(n)$  for  $R/J=1.4, 1.2, 1.0$ ; open b.c.;  $N=4900$

The exponent  $\gamma$  found is  $\gamma = 1.2 \pm 0.1$ , far from the value  $\gamma = 2$  found in the three dimensional case.

But the main problem we faced was the fact that even the system finite size has the consequence to modify a power law as the one appearing in [Eq.12] in a power law with exponential cut as [Eq.13]. In fact, the  $R/J$  value  $1.4$  that we believed to be the critical value, for simulations with  $N=900$  spins (Fig.10 ) showed to be overcritical with a greater number of spins (Fig.11 ). We then decided to observe the variation of the  $n_0$  value as a function of  $R/J$ , Fig.12 .

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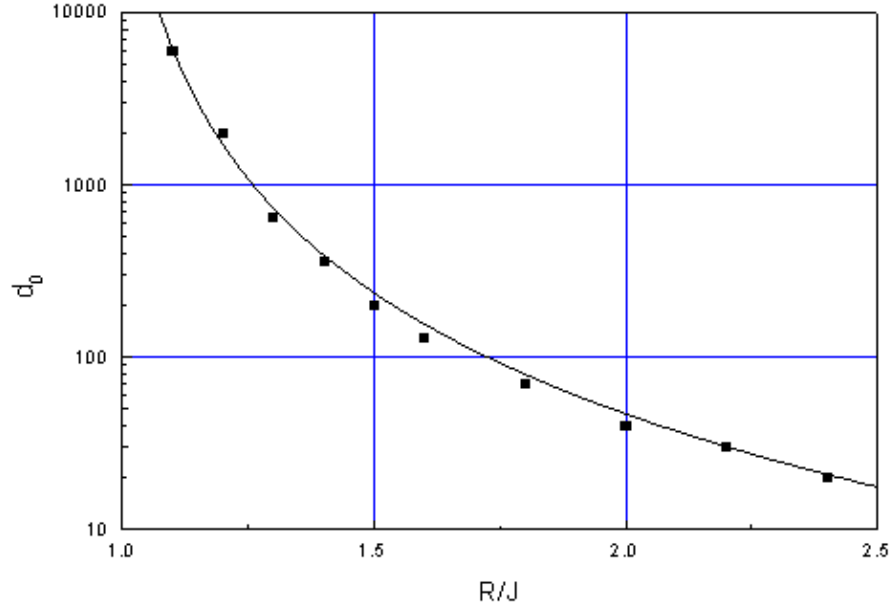


Fig.12 Variation of the  $n_0$  parameter as a function of disorder. Also shown the curve  $53(x-0.95)^{-2.5}$

The reason is that if a critical point  $R_c$  exists, then the correlation length has to go to infinity, so a divergence should appear in the  $n_0(R/J)$  behaviour. As can be observed in Fig.12, a possible fit of the data is

$$n_0 \propto \left( \frac{R}{J} - \frac{R_c}{J} \right)^{-2.5} \quad [14]$$

showing a divergence at  $R_c=0.95 J$ . But this is clearly just an indication, and a proof of the criticality of the system will have to be found with analytical methods. It is even possible that the two dimensional system do not present a critical point [Imry, Ma 1975].

### Energy landscape along the hysteresis loop

With this surely cryptic title we want to introduce the study of the characteristics of the energy landscape that the system "sees" as the state moves under the driving force of the external field. We soon found that the random properties of the RFIM model did not allow an analytical solution for the dependence of the energy on the magnetization  $E(M)$ , but we found that some approximation can be obtained in the high disorder limit.

Let us suppose the system state is changing under the external field from negative saturation to positive saturation, and that  $R \gg J$ . We already discussed this case, observing that the single spins flip almost independently, because the local random field distribution is too wide to allow the case  $|h_i^c - h_j^c| < 2J$ . In this limit, the hamiltonian [Eq.3] can be approximated in the following way. The first term becomes:

## Random Field Ising Model

$$-\frac{J}{2} \sum_{\langle ij \rangle} s_i s_j \cong -\frac{J}{2} \sum_j s_j \sum_{\langle j \rangle} s_j \cong -\frac{J}{2} NI \cdot 4I = -2JM^2 \quad [15]$$

The random field term, when the system is saturated, is nearly null, because every spin  $s_i$  is constant ( $+1$  or  $-1$ ) and the random field distribution has zero average. When the system departs from saturation, the contribution of the  $-\sum_i s_i h_i^c$  term is non null. We then approximate it with:

$$\begin{aligned} -\sum_i s_i h_i^c &\rightarrow -2N \int_{-4J-H}^{+\infty} dh^c h^c P(h^c) = \frac{-2N}{\sqrt{2\pi\sigma^2}} \int_{-4J-H}^{+\infty} dh^c h^c \exp\left(\frac{-h^{c^2}}{2\sigma^2}\right) = \\ &= -N \sqrt{\frac{2}{\pi}} \sigma \exp\left(-\frac{(4JI+H)^2}{2\sigma^2}\right) \end{aligned} \quad [16]$$

The lower limit in the integral is justified in terms of a mean field approximation, taking into account the changes in the configurations in the neighbourhood of the spins as the external field changes. In this approximation the internal field becomes  $h_i = 4JI + h_i^c + H$ .

The spin energy is then:

$$E/N \approx -2JI^2 - \sqrt{\frac{2}{\pi}} \sigma \exp\left(-\frac{(4JI+H)^2}{2\sigma^2}\right) \quad [17]$$

And it is possible to obtain the hysteresis loop shape in this approximation:

$$I\left(\frac{H}{\sigma}\right) = -1 + \frac{2}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{H/\sigma} dx \exp(-x^2/2) \quad [18]$$

Since we need the inverse expression  $H(M)$ , we will express it as  $H = \sigma F(M)$ , as the radix of [Eq.18]. Once it is substituted in [Eq.17], we obtain:

$$E/N \approx -2JI^2 - \sqrt{\frac{1}{\pi \ln 2}} R \exp\left[-\frac{F(I)^2}{2} - 8\left(\frac{\sqrt{2 \ln 2} J}{R}\right)^2 I^2 - 4\frac{\sqrt{2 \ln 2} J}{R} IF(I)\right] \quad [19]$$

that is the final approximation obtained, in the limit  $R \gg J$ .

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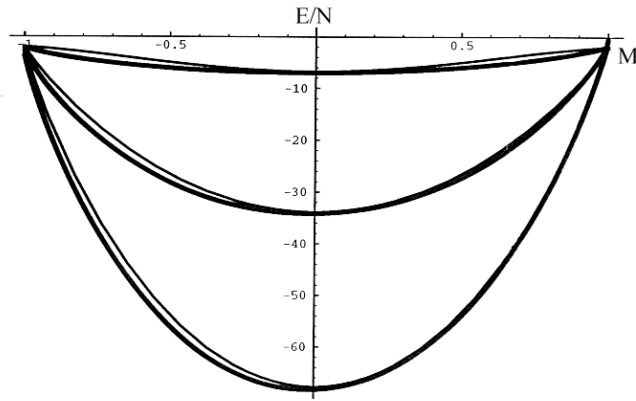


Fig.13 Energy as a function of the system magnetization. Simulation results (bold lines) and analytical form (thin lines). The disorder parameter values are  $R/J=10,50,100$

Its form is shown in Fig.13 , for  $R/J=10,50,100$ , together with the simulation results. It seems clear that the approximation leads to good results, and the deviation between the simulations and [Eq.19] seems to be due to the limited system size, that gives some non homogeneity in the set  $\{h_c^i\}$ .

The form of [Eq.19] clearly shows a higher stability for the system states with lower magnetization.

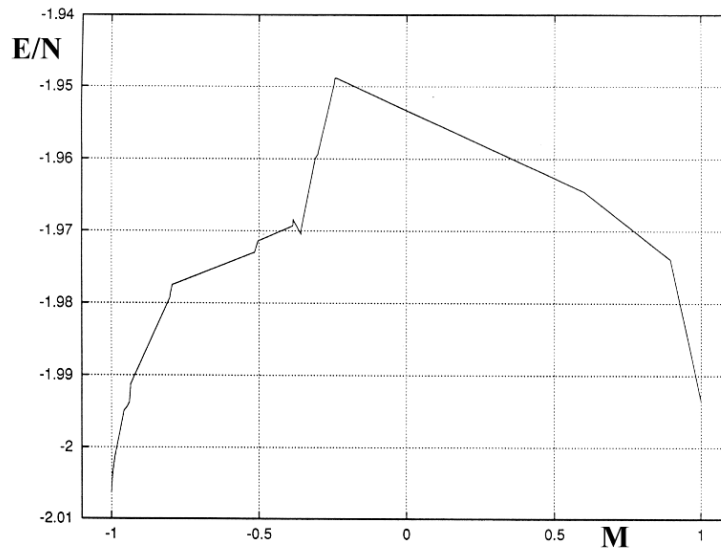


Fig.14 Simulations results for  $R=J$  showing the lower stability of the  $M=0$  state.

The limit  $R \ll J$  is more difficult to obtain in analytical form, because the interaction term in [Eq.3] is dominant, and the system behaviour results to be discontinuous, as already mentioned.

## Random Field Ising Model

In any case, the simulation results in Fig.14 show a clear peak at  $M=0$  indicating the saturated states as the more stable states of the system. In the limit case of  $R/J \rightarrow 0$  (Ising case) the only possible states are the saturated states.

### The demagnetized state

The demagnetized state is usually defined as the zero magnetization state that can be obtained when applying a particular field history. This is usually – both experimentally and in the simulations – an oscillating field with decreasing amplitude, until  $H=0$  (Fig.15 ). No proof has been found yet about the possibility that the demagnetized state *coincide* with the lowest energy state of the system. The use of a field history such as that described seems in fact to slowly select the deeper energy wells, and if the amplitude decrease is slow enough the final state could be the lowest energy state. Please note that in Fig.15 just 30 field reversal were performed to allow a clear visualisation, so the final state is very far from a  $M=0$  condition. The number of steps usually performed was higher, in any case more than a thousand of steps.

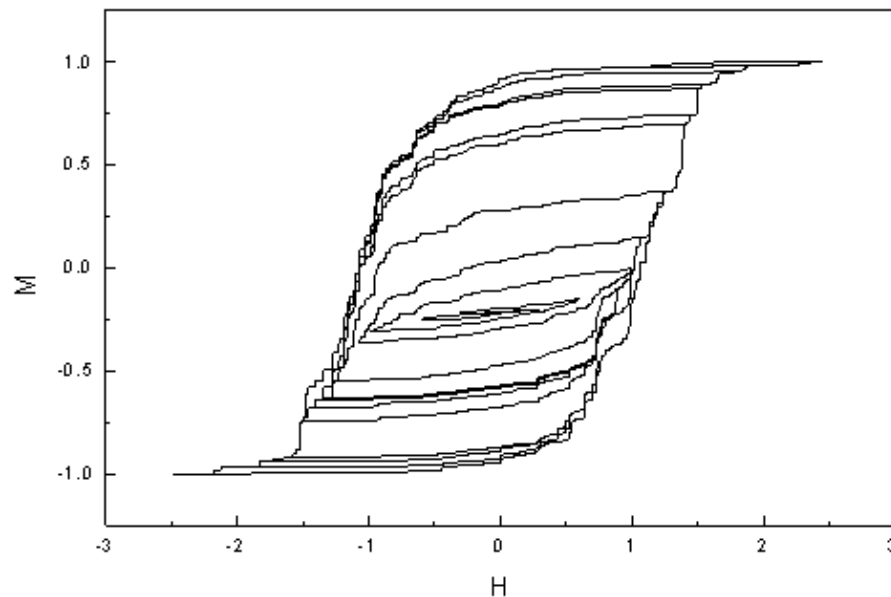


Fig.15 Hysteresis loop under oscillating external field

It must be observed that it is not always possible to reach the demagnetized state, as defined in this way. When the disorder is too low, at  $R \ll J$ , we already discussed and shown in Fig.14 that the saturated state is much more stable when compared to the lower magnetization states.

## Random Field Ising Model

### Energy

In Fig.16 the energy of the successive states assumed by the system under a demagnetizing field is shown as a function of the magnetization, for different  $R/J$  values. The states considered are the ones assumed by the system at each reversal point of the applied field.

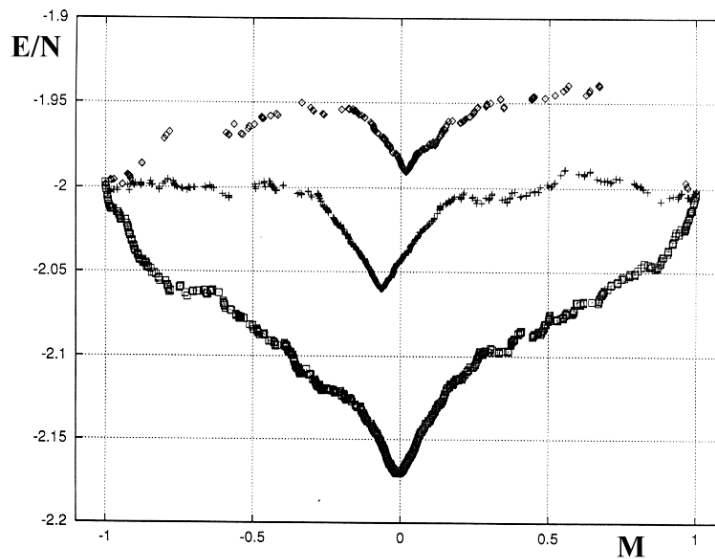


Fig.16 Energy per spin of the system states, during the demagnetization procedure, for  $R/J=2.0$  (squares),  $1.6$  (crosses),  $1.2$  (diamonds)

We can at first observe that the system states available to the system are fewer when the disorder is low, and increase as the  $R/J$  ratio increases. For  $R/J$  values lower than  $1.2$ , the demagnetization simply proved not to be possible. Moreover, we can see that the increased disorder sets the level of the demagnetized state to an energy lower than the energy of the saturated states. This minimum of the energy becomes better characterized for  $R/J > 2$ . When  $R/J > 10$ , this becomes the only minimum. The greater asymmetry of the curves at low disorder are explained with the fact that the avalanches are not symmetrically distributed when  $R/J$  is low.

The final energy after the demagnetization procedure is shown in Fig.17 , Fig.18 :

# Random Field Ising Model

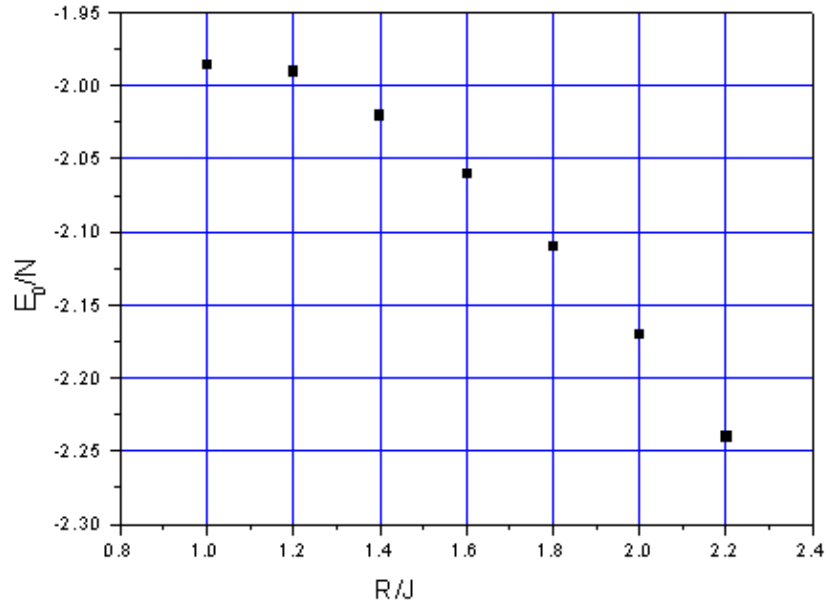


Fig.17 Demagnetized state energy per spin, as a function of  $R/J$ , low  $R/J$  values

## Random Field Ising Model

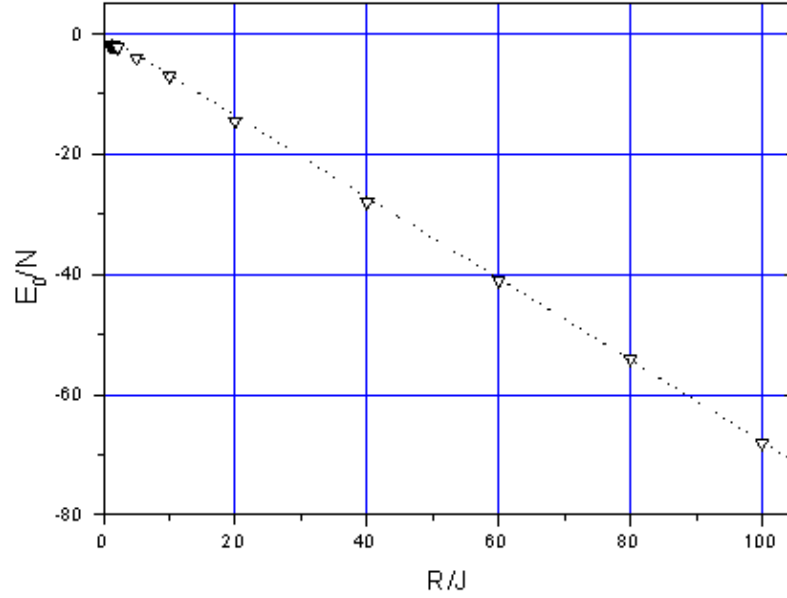


Fig.18 Demagnetized state energy per spin, as a function of  $R/J$ , high  $R/J$  values, and linear fit with [Eq.21]

Some calculation can be analytically performed to obtain the energy per spin of the state characterized by  $H=0, M=0$ . In the high disorder limit we can use in the hamiltonian

$\mathbf{H}$  the approximation used in [Eq.15] for the interaction term. The local random term can instead be approximated with:

$$-\sum_i s_i h_i^c \rightarrow -2N \int_0^{\pi} dh^c h^c P(h^c) = -N \sqrt{\frac{2}{\pi}} \sigma \quad [20]$$

so that the energy per spin of the state  $H=0, M=0$  is:

$$E_0 / N \approx -\sqrt{\frac{2}{\pi}} \sigma = -\sqrt{\frac{1}{\pi \ln 2}} R / J \quad [21]$$

Topology of the final state: fragmentation index

On the basis of the images of the two dimensional system, we also tried to characterise the demagnetized state with the study of his fragmentation. It was soon clear, by direct observation of the final states, that the demagnetized states were in fact very different from each other, depending on the  $R/J$  value. A parameter was defined ( $f$ : fragmentation index) able to discriminate between two opposite tendencies. In fact, when the constraint  $M=0$  is imposed, there are two opposite ways to obtain it: one is to have just two blocks of opposite magnetization and equal area, while the other consist in a checkerboard pattern, where spin neighbours are in

## Random Field Ising Model

opposite states. The two tendencies can be measured by a measure of the total boundary length  $B$  of the clusters present. This sum has to be considered in lattice units, and it is a count of the length of the boundaries between regions with opposite spins.

$$f = \frac{B - 2N}{2N(N-1)} \quad [22]$$

In the case of just two blocks of spins,  $f=0$ , while in the checkerboard case we have  $f=1$ .

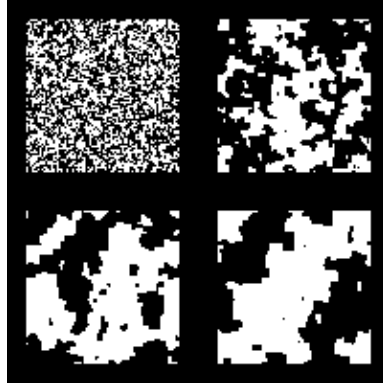


Fig.19 Four demagnetized states with (from top to bottom, from left to right)  $R/J=10.0, 2.0, 1.6, 1.4$ . Black dots are spins down, white dots are spins up

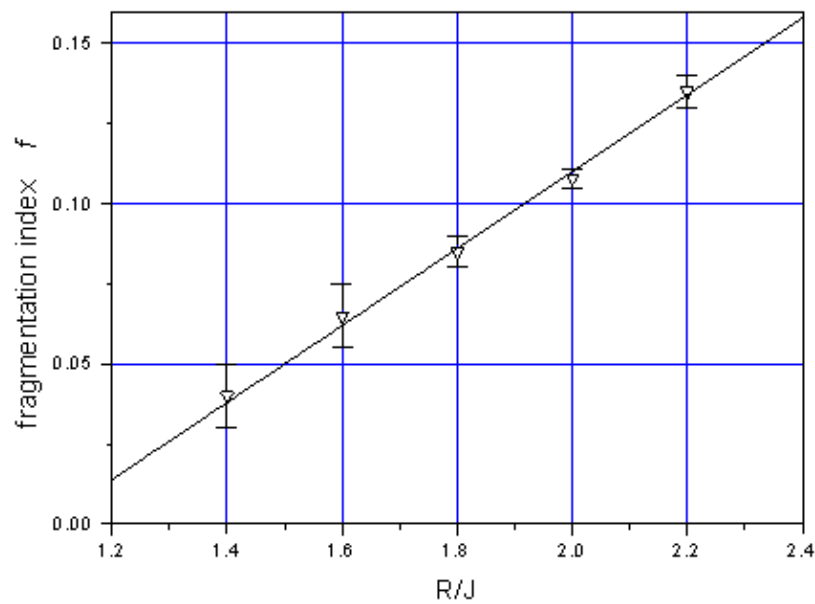


Fig.20 Fragmentation index as a function of the disorder

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We have observed (Fig.19 , Fig.20 ) that the system demagnetized state presents a high fragmentation when the disorder allows it, while for low disorder few big clusters are found, and the fragmentation is low. A linear law of the type  $f \propto 0.12 \cdot (R/J)$  seems to be followed fairly well.

### Distribution of the local energy minima in the configuration space

A possible way to analyze the energy landscape of the RFIM model could be to completely neglect the notion of attainability of a state – via a particular magnetic field history – and to pose some question about the energy of *any* stable state of the system. How are the local minima distributed in the configuration space? Is there some relationship between their relative position in this space and their energy? An unexpected help comes from the evolutionist studies of S.A. Kauffman. In his work [Kauffman 1993; Stauffer 1989; Weinberger 1991] he tried to abstractly define a model that could help him to analyze the fitness landscape (the analogous of our energy landscape, although in evolutionist studies the fitter states are at the maximum height, not at the lowest) of the protein space. Proteins being linear combinations of up to twenty aminoacids, their space is huge: for a length  $N$  protein, there exist  $20^N$  states possible, with  $19 N$  one–mutant–neighbours for each state. The  $NK$ –model was invented to analyze this space. The symbol  $N$  represents the number of "parts of the system" (aminoacids in this case), while  $K$  reflects the amount of coupling in the system (the richness of epistatic interactions, here). Without entering further in the depth of this subject, it is sufficient here to note that the  $NK$ –model was found to be deeply similar to spin glasses – better, it was found to *be* a form of spin glass. This was sufficient to us, to consider if some analysis introduced in the study of this model could be helpful in the RFIM model.

## Random Field Ising Model

What Kauffman discovered was that when the epistatic interactions were low, the local fitness optima were not randomly distributed in the genotype space, but resided near to each other. The vicinity of two states were obtained with the so-called *Hamming distance*, and we found that this measure has an exact equivalence in the overlap function we defined in [Eq.11]. Thus we performed a series of simulations, at different degrees of disorder.

First we calculated the energy of the demagnetized state  $E_0$ , with the highest number of field reversal we could afford, trying to obtain the best results. Then, a high number of random states was generated. Being these states random, that is being the set  $\{s_i\}$  independent from the internal fields  $\{h_i\}$ , these states were highly unstable, and we allowed them to relax to the nearest stable state, in zero external field. Then we calculated their energy  $E$  and the overlap with the demagnetized state. In Fig.21 are reported the results

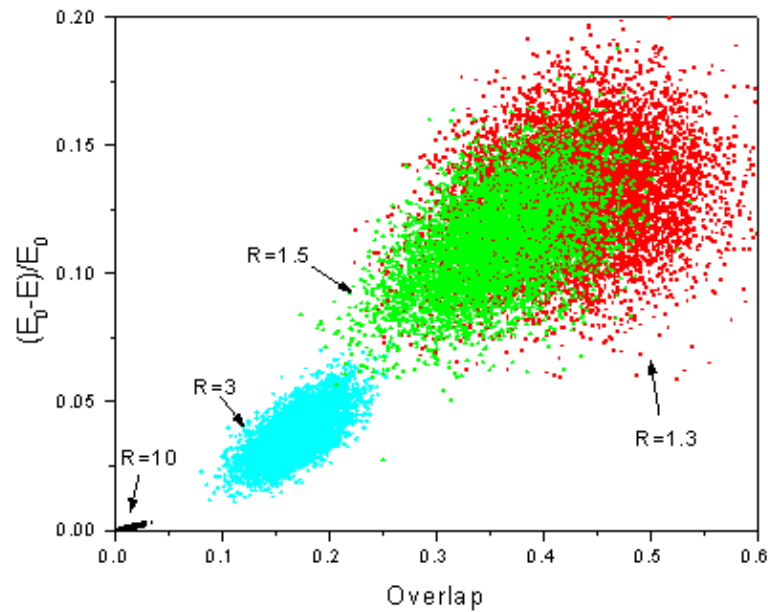


Fig.21 Relationship between the energy of locally stable states and their overlapping

This surprising result confirms what was already found from Kaufmann: there appears a global structure in the energy landscape. Let us recall that, the starting states having been randomly generated, the average magnetization should be null. Moreover, if the locally stable states were distributed randomly in the configuration space, the average overlap should be  $0.5$ . Instead this situation of uncorrelated energy landscape appears only at low disorder: in the figure, at  $R=1.3$ .

## Random Field Ising Model

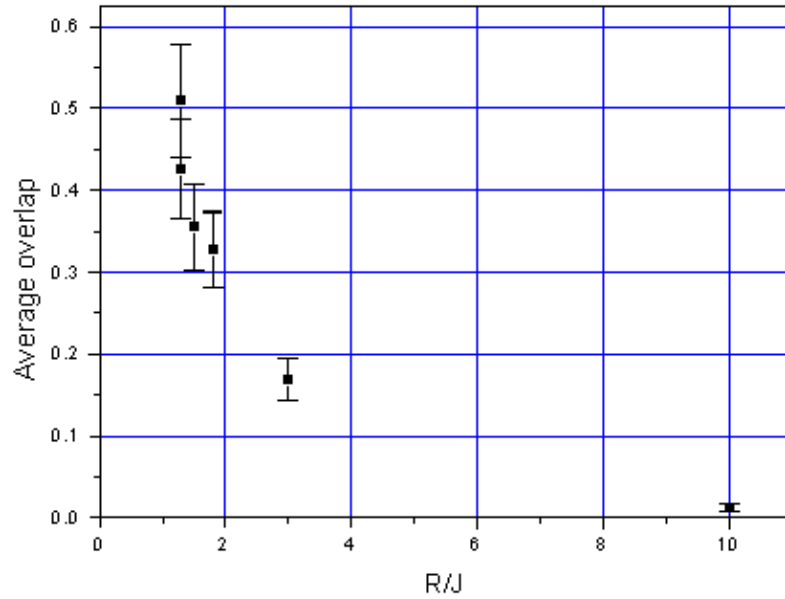


Fig.22 Average overlap for different disorder values

As the disorder increases, we see that a common shape emerges: the states are distributed in an elliptical shape. We also can observe that the distance from the demagnetized state changes: at high disorder the average overlap with the demagnetized state is smaller, and increases with the disorder. In Fig.22 this change is represented with the aid of the average overlap measure. Another feature clearly visible is that at higher disorder is greater the concentration of the points inside this ellipse, while at low disorder the dispersion is greater. This feature is clearly visible by looking at the extension of the *clouds* in Fig.21 , and also from the width of the error bars in Fig.22 . Last, and more important, is an annotation about the inclination of the ellipse's major axis. This axis seems – for all the clouds that appear to have a recognizable elliptical shape – to pass through zero and to have a positive inclination. The meaning is clear: deeper energy wells are nearer to each other, and as the distance from these more stable states increases, so the energy increases. This is the feature more assuring that a global structure in the energy landscape exists, and is more marked in the high disorder limit. It confirms the existence of something similar to an *attraction basin*, as if a low energy stable state had information about where other states with lower energy are. The states between two low energy states (*between* in the configuration space) are good candidates in the search for low energy states. Conversely, when the disorder is low, there appears to be a great scattering of local energy minima, deprived of a relationship with each other. This situation should of course regress as the disorder is really low ( $R \ll J$ ): in this case, random states would almost certainly relax towards the saturated state and, still being  $E - E_0$  great, the states should concentrate in two regions, two small clouds centered on the saturated states.