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Organization and energy properties of metastable states for the random-field Ising model

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A R T I C L E I N F O

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A B S T R A C T

Random-field Ising model (RFIM) systems are characterized by a large number of metastable states corresponding to local minima of the system energy with respect to single spin flip. We classified the minima in a hierarchical way based on the possibility of a given state to escape from a basin of mutually reachable states. We investigate the energy properties of the metastable states in relation to the basin they belong to: states of particularly high energy, obtained by fast-quenching randomly initial spin configurations, tend to have access to a complex structure of correlated basins, opposite to what is found for low-energy states. The purpose of this paper is to investigate the connection between the properties of the basin oriented graph and the energy of the corresponding states.

1. Introduction

The random-field Ising model (RFIM) has been applied in the recent years to the study of a wide array of problems, such as hysteresis in ferromagnets and first-order phase transitions. In this model, hysteresis is the result of the competition among the short-range (exchange) coupling and the interaction with frozen disorder. These terms are dominant in the behavior of a variety of different physical systems. The RFIM Hamiltonian is written in the form

$$\mathscr{H}(\{s_i\},h) = -\frac{J}{2}\sum_{\langle ij\rangle} s_i s_j - \sum_{i=1}^{N} f_i s_i - \sum_{i=1}^{N} h s_i, \tag{1}$$

where the system is defined by N Ising spin, $s_i = \pm 1$ on a d dimensional lattice having periodic boundary conditions. Each spin interacts with its $2d$ first-neighbors spins with exchange coupling ($J>0$), and is also coupled with the external field h and with a random field f_i, Gaussian distributed with zero mean and variance σ^2. To each configuration $\omega = \{s_i\}$ corresponds a value of the order parameter, the magnetization m: $m = 1/N\sum_i s_i$. The ratio σ/J controls the order–disorder relationship: assuming $J=1$ it will be sufficient to consider only the value of σ. By lowering the amount of disorder σ, if $d \geqslant 3$, the system undergoes a phase transition at σ_c passing from a condition where the interaction dominates to one where the disorder is predominant [1]. In the following, we will investigate the system just in the $\sigma > \sigma_c$ case.

The zero temperature study of hysteresis is performed here by considering the out-of-equilibrium dynamics, in which the system evolution is achieved by varying the external field and applying the single-spin flip dynamics to the spin configuration. Under these conditions, the model exhibits return-point memory [1]. The number of possible spin configurations $\mathscr{C}$ is 2^N; a state is named *stable* when it is an energy minimum with respect to any single-spin flip. Stable states are only a subset, which we term $\mathscr{S} \subseteq \mathscr{C}$, of all possible states. The stability condition that has to be satisfied by any stable state is that each spin is aligned to the corresponding internal field: $s_i = \mathrm{sign}(h_i)$, where

$$h_i = J\sum_{\langle j\rangle_i} s_j + f_i + h. \tag{2}$$

When the dynamics is such that the state $\mathbf{s} = (\omega, h)$ is driven by the field h, the evolution of h leaves the configuration ω unchanged, until a field h' where the stability condition is no more satisfied. At that point, the system becomes unstable, an avalanche of spin-flips occurs and the system reaches a new configuration ω' and state $\mathbf{s}' = (\omega', h')$.

2. Basins structure of the energy landscape

Starting from one of the saturation states $\mathbf{s}_\infty^{\pm}$, and driving the system by varying the external field, we are able to explore a particular set of stable states $\mathscr{R} \subseteq \mathscr{S}$, characterized by the fact that they are all reachable from $\mathbf{s}_\infty^{\pm}$ by the application of some–however complex—field history. However, not all stable states are reachable from saturation, but only a fraction. The size of $\mathscr{R}$ as a function of N grows more slowly than the size of $\mathscr{S}$, so

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that in the thermodynamic limit the relative number of field reachable states will be negligible with respect to the number of possible stable states [3]. To better explore the hierarchy of system states, we have therefore to investigate the connections between states which are not field reachable.

We proved [2] that *all* stable states $\mathbf{s} \in \mathcal{S}$ are organized into equivalence classes of mutually reachable states, termed *basins*. A basin B is a subset of $\mathcal{S}$ for which, given any two states belonging to B, two field histories connecting the given states always exist. The set $\mathcal{R}$ is the particular basin containing the two saturation states $\mathbf{s}^{\pm}_{\infty}$. Each basin B_i is represented by a pair of limit states $\mathbf{s}^{\pm}_i$ (*twin states*) at field $h^{\pm}_i$, which are the only exit points from the given basin (if the basin is $\mathcal{R}$, the twin states are the two saturation states $\mathbf{s}^{\pm}_{\infty}$, and $h^{\pm}_i \to \pm\infty$). The sequence of stable states obtained going from one twin to the other (*basin-loop*) is the corresponding concept of the major hysteresis loop in $\mathcal{R}$.

It is evident that the notion of *basin* allows to generalize the notion of the set of field reachable states. Starting from a state $\mathbf{s} \in B_i$, the state continues to belong to B_i as long as any applied field history does not force the state beyond the twin states of the basin $\mathbf{s}^{\pm}_i$. It is possible to verify that for any $\mathbf{s} \in B_i$ a partition exists between *active* spins and *inactive* spins. Only the active spins ω_i are able to flip as the state $\mathbf{s}$ evolves, while the inactive spins remain frozen: those spins would flip under fields outside the interval (h^-_i, h^+_i). If the basin is $\mathcal{R}$, all the spins are active. As the state $\mathbf{s}$ reaches the twin state $\mathbf{s}^+_i$ ($\mathbf{s}^-_i$), all the active spins ω_i are saturated in the up (down) direction. A change in the inactive spins set is visible only after the field goes beyond $h^{\pm}_i$. Entering into a new basin B_{i+1} is equivalent to the activation of a new cluster of spins ω' ($\omega_{i+1} = \omega_i \oplus \omega'$), previously inactive. If we enter at increasing field, for example, the new twin state field h^+_{i+1} is such that ω' will be saturated up. It can be proven instead that the lower field is $h^-_{i+1} \leqslant h^-_i$. Therefore, the relationship $h^-_{i+1} = h^-_i$ will hold, until ω' will be contiguous to a portion of ω_i that reversed at the field h^-_i, decreasing its value. The probability of two subsets of active spins of being contiguous at low $|M|$ values is negligible, therefore, as shown in Fig. 1, with increasing (decreasing) field, the field h^-_i (h^+_i) remains constant for a number of successive basins.

Using the active/inactive spin picture, we see that–once inside a basin—the system behavior can be mapped to a RFIM system composed just by the active spins component, while the inactive spins behave as a disordered boundary condition. Each successive basin will span a larger field interval (including the preceeding one $h^-_{i+1} \leqslant h^-_i < h^+_i \leqslant h^+_{i+1}$), and a larger magnetization interval.

The hierarchy of basins is connected, so that starting from a given basin B_0, it is possible to visit a sequence of basins before reaching $\mathcal{R}$. But the connection is one way only. Every time the system exits from a basin B_i through one of its twin states into a new basin B_{i+1}, there is no field history bringing the system back to B_i (if it existed, the basin B_{i+1} would coincide with B_i). Basins are organized as a binary oriented graph, with the basin $\mathcal{R}$ of reachable states as the bottom of the graph.

Newman and Stein explored this *diode effect* in [4] for a variety of models exhibiting broken ergodicity. They observed that for a class of models the transitions between different subsets of available states (basins) happen in a one-directional way, originating a graph structure that emerges naturally from the dynamics, although the models themselves do not show any hierarchical structure. In this sense, the $T = 0$ RFIM model is perfectly apt to explain this class of behaviors.

Due to the complex graph structure described, given a starting state in a basin B_0 and the application of an external field as the only dynamics allowed, it is clear that it will be possible to explore only the subgraph having the root on B_0 and ending in $\mathcal{R}$, since the dynamics will never allow to jump into a different, parallel subgraph. One interesting feature of this system [5] is that the size of this subgraph, i.e. the span of basins that can be explored starting from B_0, is lower if the starting state is the ground state (GS). The measure used for the graph size is the shortest path D from B_0 to $\mathcal{R}$ (critical path), whose average value depends on disorder as $\langle D \rangle \sim 1/\sigma$. Our analysis further demonstrated that more generally the average size of the subgraph is related to the starting state energy. A first and very striking insight into this property is that the energy values of the reachable and the non-reachable states are on the average very different [3] (see Fig. 2): reachable states average energy is lower with respect to non-reachable states. This, however, does not necessarily mean that the GS is field reachable: there is a definite probability, increasing with disorder and decreasing with N, that it *can* be reachable.

We can imagine that a non-field-reachable state, as it evolves under the field dynamics, will decrease its free energy as it moves along the graph toward the bottom basin $\mathcal{R}$. In Fig. 3 we show the basins visited starting from a random stable state (RND) and from the GS. The most interesting difference when exiting from the basin containing a RND or the GS is the energy behavior of the curve of the basin remanences $r_{RND}(M)$ and $r_{GS}(M)$. The curve

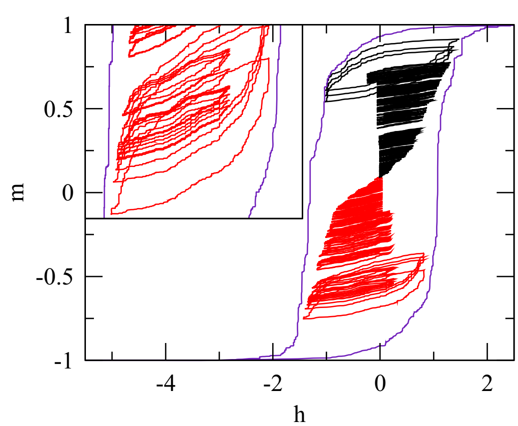

Fig. 1. The basin loops obtained when starting from a random stable state, for field histories monotonically increasing (black) and decreasing (red). Inset: decreasing branch basin loops enlargement. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

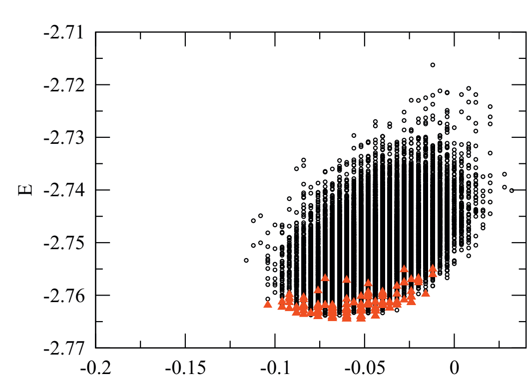

Fig. 2. Plot of the energy values of a large number (30,000) of stable states at fixed field $h = 0$. In the lower part of the graph we find the reachable state (red triangles). $d = 1$, $N = 500$, $\sigma = 3$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

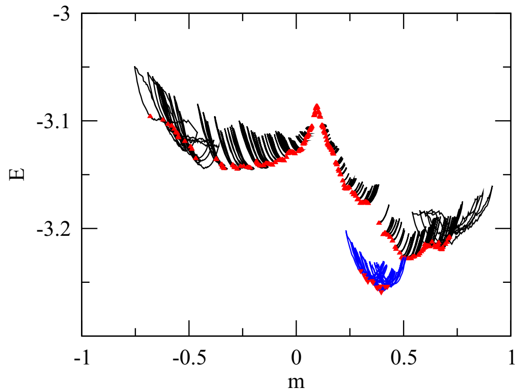

Fig. 3. Basins structure when exiting from the ground state (blue lines at $m>0$) and from a random state (black lines) for $d=3$, $N=15^d$ and $\sigma=3$. Red down (up) triangles indicate each basin remanence energy for the ground state (random state) basins. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$r_{GS}(M)$ increases with $|M|$ as it leaves the GS, a sure sign of the increasing energy that is found as soon as one leaves the GS basin. This increase continues until the state reaches $\mathscr{R}$, if the GS does not already belong to it. On the contrary, the curve $r_{RND}(M)$ exhibits a non-monotonous behavior: a sudden decrease in energy, followed by a successive increase. The initial decrease of energy is correlated on one hand on the fact that a typical RND can be considered as having been generated by a fast quenching procedure—followed by a stabilization to the nearest energy minimum: its average energy will be considerably high. On the other hand, the field dynamics moves this state in the direction of states belonging to $\mathscr{R}$, having a lower average energy. When the field increases, the change in energy cannot be overcome and the contribution related to the external field is predominant. That explains the behavior at large field values.

Our analysis of the energy landscape of the RFIM showed therefore a complex, hierarchical behavior. We believe that the observed features could be of extreme interest in the investigation of this model at finite temperature, since the irreversibility of the motion when exiting from high energy basins has been shown [4] to hold even in the $T\neq0$ case for many similar models.

References

[1] Sethna, P. James, Dahmen, Karin, Kartha, Sivan, et al., Hysteresis and hierarchies: dynamics of disorder-driven first-order phase transformations, Phys. Rev. Lett. 70 (1993) 3347.
[2] G. Bertotti, P. Bortolotti, A. Magni, V. Basso, Topological and energetic aspects of the random-field Ising model, J. Appl. Phys. 101 (2007) 09D508.
[3] V. Basso, A. Magni, Field history analysis of spin configurations in the random field Ising model, Physica B 343 (2004) 275–280.
[4] D.L. Stein, C.M. Newman, Broken ergodicity and the geometry of rugged landscapes, Phys. Rev. E 51 (1995) 5228–5238;
D.L. Stein, C.M. Newman, Spin-glass model with dimension-dependent ground state multiplicity, Phys. Rev. Lett. 72 (1994) 2286–2289.
[5] A. Magni, V. Basso, Study of metastable states in the random field Ising model, J. Magn. Magn. Mat. 290–291 (2005) 460–463.