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Nonlinear aspects of Landau–Lifshitz–Gilbert dynamics under circularly polarized field

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Abstract

Landau–Lifshitz–Gilbert dynamics of a particle under circularly polarized field is reduced to exactly solvable autonomous form. Two or four rotating magnetization modes are possible. Stability, bifurcation diagrams, and existence of quasi-periodic modes are discussed. Connection with Stoner–Wohlfarth theory is shown.

Keywords: Magnetization—time dependent; Landau–Lifshitz equation; Stoner–Wohlfarth model; Ferromagnetic resonance

Landau–Lifshitz–Gilbert (LLG) equation has been the focus of considerable interest in connection with ferromagnetic resonance and, more recently, spin dynamics in magnetic recording media [1,2]. Various types of nonlinear phenomena have been studied in relation to the excitation of nonuniform magnetization modes [3]. However, less attention has been paid to the fact that many of these phenomena are predicted by LLG equation even under uniform magnetization. In this work, the case is studied of a uniaxial particle uniformly magnetized and subjected to a circularly polarized field [4]. Rotational invariance about the particle axis permits one to reduce the problem to the study of an autonomous dynamical system exactly solvable in analytical terms.

We consider a uniformly magnetized spheroidal particle with symmetry axis along z. LLG equation reads, in dimensionless Gilbert form

$$\frac{\mathrm{d}\boldsymbol{m}}{\mathrm{d}t} = -\boldsymbol{m}\times\left(\boldsymbol{h}_{\mathrm{eff}} - \alpha\frac{\mathrm{d}\boldsymbol{m}}{\mathrm{d}t}\right), \quad (1)$$

where $\boldsymbol{h}_{\mathrm{eff}} = \boldsymbol{H}_{\mathrm{eff}}/M_{\mathrm{s}}$, $\boldsymbol{m} = \boldsymbol{M}/M_{\mathrm{s}}$, time is measured in units of $(\gamma\mu_0 M_{\mathrm{s}})^{-1}$, $\alpha > 0$, M_{s} is the saturation magnetization, and γ the gyromagnetic ratio, taken as positive. The applied field rotates in the x–y plane at the angular frequency ω and has a constant component along z. Accordingly, $\boldsymbol{h}_{\mathrm{eff}} = (h_{a\perp}\cos\omega t, h_{a\perp}\sin\omega t, h_{az} + \kappa_{\mathrm{eff}} m_z)$, where the effective anisotropy constant κ_{eff} summarizes shape and crystal anisotropy effects. Let us pass to the rotating frame where the applied field is stationary, and let us set $m_x = \sin\theta\cos(\omega t - \phi)$, $m_y = \sin\theta\sin(\omega t - \phi)$, $m_z = \cos\theta$. The angle ϕ represents the magnetization lag with respect to the rotating field. In terms of (θ, ϕ) Eq. (1) becomes

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} - \alpha\sin\theta\frac{\mathrm{d}\phi}{\mathrm{d}t} = \kappa_{\mathrm{eff}}[b_\perp\sin\phi - \Omega\sin\theta], \quad (2)$$

$$\alpha\frac{\mathrm{d}\theta}{\mathrm{d}t} + \sin\theta\frac{\mathrm{d}\phi}{\mathrm{d}t} = \kappa_{\mathrm{eff}}[b_\perp\cos\phi\cos\theta - (b_z + \cos\theta)\sin\theta], \quad (3)$$

where $b_z = (h_{az} - \omega)/\kappa_{\mathrm{eff}}$, $b_\perp = h_{a\perp}/\kappa_{\mathrm{eff}}$, $\Omega = \alpha\omega/\kappa_{\mathrm{eff}}$. Differences between systems are measured by $(\alpha, \kappa_{\mathrm{eff}})$, differences in the excitation conditions by $(b_z, b_\perp, \Omega)$ or, equivalently, $(h_{az}, h_{a\perp}, \omega)$.

Eqs. (2) and (3) describe an autonomous dynamical system on the unit sphere. The autonomous formulation,

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made possible by the rotational symmetry of the problem around the z axis, permits one to draw some general conclusions about the equilibrium points (EP) of the dynamics. We recall that EPs refer to the rotating frame, so in the laboratory frame they will represent magnetization modes rigidly rotating with the field. According to Poincaré index theorem [5], the number of nodes and foci minus the number of saddles of any autonomous dynamics on the sphere is equal to two. Therefore, LLG dynamics under rotating field will have at least two EPs, of node-type or focus-type. As a matter of fact, the number of EPs is limited to two or four. This can be seen by setting $d\theta/dt = d\phi/dt = 0$ in Eqs. (2) and (3) and by eliminating ϕ from the resulting equations. One finds that EPs obey the following fourth-order polynomial equation in $m_z = \cos\theta$:

$$\frac{b_\perp^2}{1-m_z^2} - \frac{(b_z+m_z)^2}{m_z^2} = \Omega^2 \tag{4}$$

which will have at most four real roots in the interval $-1 \leqslant m_z \leqslant 1$. Therefore, two EP configurations are possible: (i) two nodes or foci; (ii) three nodes or foci plus one saddle.

The stability of EPs is controlled by the trace and the determinant of the stability matrix A obtained by linearizing Eqs. (2) and (3) around a given EP. By standard methods [6] one obtains

$$\operatorname{tr} A = -\frac{2\alpha\kappa_{\text{eff}}}{1+\alpha^2}\left[\left(1+\frac{b_z}{m_z}\right) - \frac{1-m_z^2}{2} + \frac{\Omega m_z}{\alpha}\right], \tag{5}$$

$$\det A = \frac{\kappa_{\text{eff}}^2}{1+\alpha^2}\left[\left(1+\frac{b_z}{m_z}\right)^2 - (1-m_z^2)\left(1+\frac{b_z}{m_z}\right) + \Omega^2 m_z^2\right]. \tag{6}$$

Stable nodes or foci are characterized by ($\operatorname{tr} A < 0$, $\det A > 0$), unstable ones by ($\operatorname{tr} A > 0$, $\det A > 0$), saddles by ($\det A < 0$) [6]. Therefore, a saddle can exist (at fixed frequency) only inside the $(b_z, b_\perp)$ region delimited by $\det A = 0$. This boundary can be expressed in the form $[b_z(m_z), b_\perp(m_z)]$, with m_z as independent variable. From Eq. (6), one finds

$$b_z(m_z) = -m_z\left[1 - \frac{1-m_z^2}{2}\times\left(1 \pm \sqrt{1-\left(\frac{2\Omega m_z}{1-m_z^2}\right)^2}\right)\right] \tag{7}$$

Then, $b_\perp(m_z)$ is obtained by inserting Eq. (7) into Eq. (4). An example of ($\det A \leqslant 0$) region is shown in Fig. 1. Interestingly, this region appears as the dynamic generalization of Stoner–Wohlfarth (SW) astroid [7]. In fact, in the limit $\omega \to 0$ the two branches of Eqs. (7) and (4) reduce to $[b_z(m_z), b_\perp(m_z)] = [-m_z^3, (1-m_z^2)^{3/2}]$ and $[b_z(m_z), b_\perp(m_z)] = [-m_z, 0]$. The first branch is nothing but SW astroid, $b_z^{2/3} + b_\perp^{2/3} = 1$. The line $b_\perp = 0$ has no particular meaning in the $\omega \to 0$ limit. However, its dynamic generalization plays a definite role, because it forbids the existence of saddle EPs at low rotating field amplitudes.

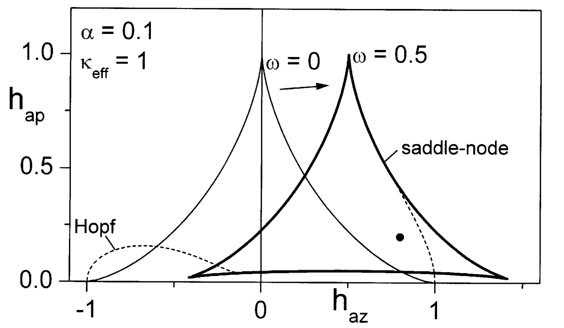

Fig. 1. Field control plane for system with ($\alpha = 0.1$, $\kappa_{\text{eff}} = 1$), at $\omega = 0.5$. Heavy solid line: saddle-node bifurcation line ($\det A = 0$); dashed line: Hopf bifurcation line ($\operatorname{tr} A = 0$ with $\det A > 0$). Thin solid line is Stoner-Wohlfarth astroid ($\det A = 0$ at $\omega = 0$). Solid point indicates location of phase portrait of Fig. 2.

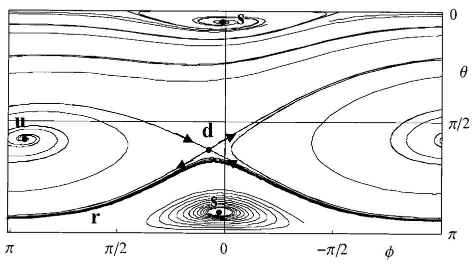

Fig. 2. Phase portrait calculated through numeric integration of Eqs. (2) and (3) for system with ($\alpha = 0.1, \kappa_{\text{eff}} = 1$) at ($h_{az} = 0.8$, $h_{a\perp} = 0.2, \omega = 0.5$) (see Fig. 1). Stable (s), unstable (u), and saddle (d) EPs are indicated. A repelling limit cycle (heavy solid line denoted by r) is also present.

The ($\det A \leqslant 0$) astroid is one among several elements that characterize LLG dynamics under rotating field. Qualitatively different phase portraits are possible, which transform into each other through different types of bifurcations. Interestingly, limit cycles are necessarily present under certain conditions. For example, when the EPs are two and are both stable or both unstable, the dynamics must admit at least one limit cycle as a consequence of Poincaré–Bendixson theorem [6]. A limit cycle is a periodic solution in the rotating frame, so it will represent a quasi-periodic magnetization mode, deriving from the combination of the rotating field and the limit cycle periods. We have found that phase portraits with two or four EPs and zero, one, or two limit cycles may occur (see Fig. 2 for an example). In this context, the condition $\det A = 0$ identifies saddle-node bifurcations, where the system passes from two to four EPs or viceversa. On the other hand, Hopf bifurcations, occurring at (tr $A = 0$, det $A > 0$) (dashed line of Fig. 1, calculated from Eqs. (4)–(6)), control the creation or destruction of limit cycles. We have found that also homoclinic saddle connections and semi-stable-limit-cycle bifurcations [6] take place, which give rise to a surprisingly rich phase portrait structure, to be analyzed in detail in future work.

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