

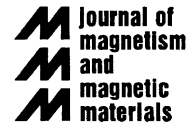


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Journal of Magnetism and Magnetic Materials 290-291 (2005) 460–463



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Study of metastable states in the random-field Ising model

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Available online 9 December 2004

Abstract

The random-field Ising model (RFIM) provides a convenient framework to investigate complex energy landscapes. We use an out-of-equilibrium $T = 0$ single spin flip dynamics induced by varying the applied field, starting from locally stable configurations $\{s_i\}$. If the configuration considered cannot be reached by a field history from saturation, it is possible to define a set of connected states which is defined as a basin. A whole hierarchy of basins is found when the field is increased outside the limits of the initial basin. The resulting structure has the topology of an oriented graph. The properties of the graph cast new light on properties of the ground state (GS) and the possibility to reach the GS by a field history from saturation. We have numerically determined the graphs in RFIM realizations of finite size in one dimension for arbitrary selected states and for the GS. Remarkable differences between them are found in the critical path of the corresponding graphs, the GS being nearer to the field reachable states than a generic state.

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PACS: 75.60.Ej; 75.10.Nr

Keywords: Random-field Ising model; Hysteresis; Graph theory

1. Introduction

In the RFIM a convenient tool to study the energy landscape is to consider the configurations obtained by out-of-equilibrium single spin flip dynamics when the applied field is changed. Thanks to partial ordering relation between states conserved by the field dynamics [1] it has been recently shown that not all locally stable configurations can be reached by an appropriate field history starting from saturation. A number of observations are then possible: the reachable configurations contain the memory of the reversal fields that generated it and the sequence of the reversal fields can be retrieved

from a stable state for a given system realization of finite size; the relative size of the set of field reachable (H-states) and locally stable (S-states) depends on the system size and on the disorder amplitude; there exists a strong correlation between the energy of the states and the possibility to be reached by a field history [2]. Much of the interest was recently concentrated on the properties of the demagnetized state (DS, obtained by an oscillating field with slowly decreasing amplitude), which is believed to be one of the lowest energy states [3]. However, recent results showed that, for the linear chain, the ground state (GS) and the DS (in the limit $N \rightarrow \infty$) have different energies, which implies that they are not the same state [4].

In this paper we have investigated the hysteresis properties in the RFIM starting from an arbitrary configuration not reachable by a field history from saturation, say s_0 stable under the field $H = 0$, in order

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to obtain information about the topology of the energy landscape and its relation with the H-states. When a field history is applied, a sequence of new configurations is generated. We found that it is possible to classify the found states into basins of mutually field-connected states. The set of basins forms an oriented graph having as a source the basin of the \mathbf{s}_0 starting state and as a sink the set of H-states. From any basin new ones may be found by increasing or decreasing the field. Examples of the graph structure were obtained numerically by considering realizations of the RFIM in one dimension at finite size under periodic boundary conditions. We present here the numerical results of the determination and analysis of the graphs obtained starting from arbitrary states and from the GS. We found, by performing a statistical analysis over different realizations of the same disorder amplitude, that the average of the maximum number of basins separating the source from the sink (the critical path) is considerably different in the two cases.

2. Basins of connected states and oriented graph structure of basins

In the RFIM the dynamics under varying field is studied by using the stability condition $s_i = \text{sgn}(h_i)$, where the internal field $h_i = J \sum_{\langle k \rangle_i} s_k + f_i + H$ is obtained by the sum over the k spins interacting with spin i ; f_i is the quenched-in random field and H the applied field. The stability condition expresses the fact that the state $\{s_i\}$ is a local minimum of the energy $\mathcal{H} = -\sum_{\langle ij \rangle} J s_i s_j - \sum_i (f_i + H)$ with respect to the inversion of a single spin, where the sum $\langle ij \rangle$ runs over the couples of interacting spins. When the field H is changed, the original starting state may become unstable and a new stability condition is found. The evolution is then a sequence of avalanches followed by H intervals where the system state does not change.

We begin by giving the definition of what is a basin of connected states and we will then proceed to the analysis of the structure of basins, which results to be an oriented graph. We limit the investigations to stable states under $H = 0$, since the results are trivially extensible at any H .

Consider a spin configuration \mathbf{s}_0 . A new state, say $\mathbf{s}_0^{(1)}$, is obtained by the application and the removal of the field H_1 . We now wonder if a field history tracing back from $\mathbf{s}_0^{(1)}$ to the initial state \mathbf{s}_0 exists. If this history exists then the two states are said to be mutually connected by the field. This mutual connection leads to the definition of a set of connected states or basin. A basin B is a set of states where any element can be obtained from any other by a field history. It is immediate to realize that the whole ensemble of H-states is a basin. In fact, any state can be connected by using the saturation as a passage state: it can be always reached by any state and any H-

state can be obtained from it. Different is the case for non H-states. Here we face the following problem: how can we determine a field history leading from $\mathbf{s}_0^{(1)}$ back to \mathbf{s}_0 given the partial ordering relation (say for positive $H^{(1)}$, $\mathbf{s}_0^{(1)} \geq \mathbf{s}_0$)? Here we recall the algorithm given in Ref. [2] that allows to obtain the sequence of reversal fields of the *minimal field history* from the saturation remanence. The algorithm still apply here where we start from $\mathbf{s}_0^{(1)}$, being the condition $\mathbf{s}_0^{(1)} \geq \mathbf{s}_0$ sufficient for its application. If, by using the algorithm, the sequence of reversal fields taking from $\mathbf{s}_0^{(1)}$ to \mathbf{s}_0 is obtained, then the states $\mathbf{s}_0^{(1)}$ and \mathbf{s}_0 belong to the same basin, say B_0 . Suppose we now repeat the process for a larger positive field $H^{(2)}$ and that, starting from the new found state $\mathbf{s}_0^{(2)}$, no field history can be obtained by the algorithm from $\mathbf{s}_0^{(2)}$ to \mathbf{s}_0 . Then the new state does not belong to B_0 and we may label it as $\mathbf{s}_1 \equiv \mathbf{s}_0^{(2)}$ as the first state of a new basin B_1 .

After this description of how states can be arranged into basins, we proceed by giving two properties of a basin without providing the proofs, these being based on the partial ordering preserved by the dynamics. (i) Each basin B is characterized by two limit states $\mathbf{s}^{(+)}$ and $\mathbf{s}^{(-)}$ such that each state of the basin $\mathbf{s} \in B$ is partially ordered with respect to them: $\mathbf{s}^{(+)} \geq \mathbf{s} \geq \mathbf{s}^{(-)}$. (ii) The two limit states are obtained from any state in B when the field $H^{(+)}$ (or $H^{(-)}$) is applied and removed. When the field exceeds the $H^{(+)}$ or $H^{(-)}$ limit fields a new basin is then found.

The ensemble of the basins forms a graph in which each basin is a vertex. The graph is ordered, since all lines have an arrowhead indicating that, by the field history, the states in different basins may be connected in one direction only. Each vertex has two exiting lines and many possible in, excluding the H-states basin which has many entrances and no exits. When a graph is formed from the state \mathbf{s}_0 the basin of the starting state is then the source and the H-states set is the sink of the resulting oriented graph. The investigation of the details of a graph structure requires then a realization of the RFIM and a starting state.

2.1. Search algorithm

The problem of identifying the graph structure in a finite system was dealt by a recursive search. The process we follow is, in a sense, symmetrical with respect to the approach used in [2]: instead of starting from the saturation and recovering a field history with which to reach the given state \mathbf{s}_0 , we start from \mathbf{s}_0 and move towards saturation, while examining all the way up if we exit from a basin of mutually reachable states. Starting from \mathbf{s}_0 , belonging to the basin B_0 , the field is increased up to the first instability at field H_1 , after which it is returned to zero to the remanence $\mathbf{s}_0^{(1)}$; the process is iterated progressively for increasing fields $H^{(1)} < \dots < H^{(i)} < \dots$ thus generating a hierarchy of

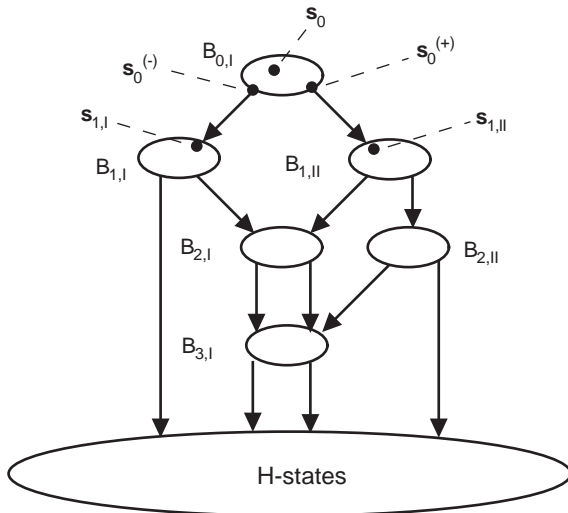


Fig. 1. Example of a graph generated from an arbitrary state s_0 .

first-order remanences $s_0^{(I)} \leq \dots \leq s_0^{(j)} \leq \dots \leq s_0^{(+)} \leq s_{1,I}$. The same process is followed for decreasing fields. $s_{1,I}$ and $s_{1,II}$ are the first states encountered in the process for decreasing and increasing fields, respectively, that do not belong to B_0 . The remanence states found by this procedure, even if they are a subset of B_0 , always include by construction the two thresholds $s_0^{(+)}$ and $s_0^{(-)}$, so we are able to find the two exit points of the basin by just investigating first-order remanences. Once we exit from B_0 the procedure continues recursively starting from $s_{1,I}$ and $s_{1,II}$ as the entry points of the basins $B_{1,I}$ and $B_{1,II}$ at the next level 1—where the first index (0, 1, ...) is the recursions index, while the second index (I, II, ...) classifies the basins at the same level (see Fig. 1). This search procedure is then structured as a binary tree: at each i -level the iteration splits in two at the $i + 1$ level for the two states found by increasing or decreasing field. In the last iteration the unique H-state set is found.

The quantity chosen to analyze the complexity of the graphs is the critical path, i.e. the maximum number of vertex, excluding loops, to be visited in order to go from the source to the sink. Having used an algorithm based on an iterated binary search the critical path results to be the number of algorithm iterations needed to go from source to sink.

3. Hysteresis properties starting from the GS and from arbitrary stable states

We compared the graphs for both randomly generated stable states and the GS as starting points. Investigations were performed numerically on finite

systems having $N = 500$ spins and different disorder amplitudes, with random fields f_i sampled from a Gaussian distribution of variance R^2 . $N_{\text{iter}} = 10$ different random (RND) stable states s_0 were generated for each of $N_{\text{dis}} = 10$ disorder realizations. The GS was found for $N_{\text{dis}} = 100$ disorder realizations. The search algorithm was applied in order to find the graph for the mentioned states. When the same realization is analyzed by starting from different initial states, the corresponding graphs may in principle overlap, since when a random state is chosen nothing can be said by our method on the existence of higher hierarchy basins leading to the basin of the starting state. As a measure of the distance of the investigated starting state from the H-states we used the critical path D of the found graph. In Fig. 2 is shown the plot of the $\langle D \rangle$, averaged over different realizations for the GS and over different starting states and realizations for random states, as a function of disorder R^2 . The dependence of the critical path D on R^2 is connected with the typical size of the graph, i.e. the number of basins found, which is in turn, proportional to the number of stable states of the system realization. This number depends on the disorder R and on the system size N . What is worth noticing in the result of Fig. 2 is the large difference between $\langle D \rangle_{\text{RND}}$ and $\langle D \rangle_{\text{GS}}$ at the same R , which is more than one order of magnitude and is systematically found for different disorders.

This difference implies that the structure of basins connecting the GS with the H-states is considerably simpler than that for randomly selected states. This observation points to the existence of a correlation between the energy of the states in a basin and their

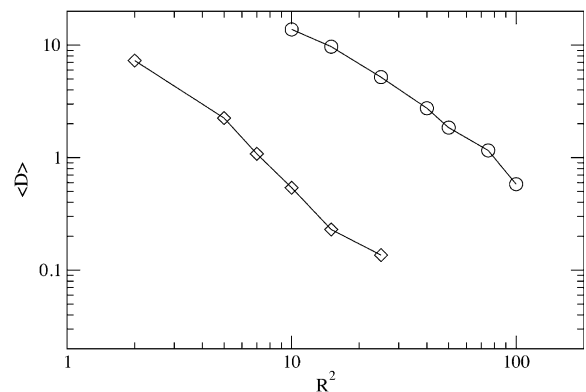


Fig. 2. Size of graph for finite systems. The picture shows the dependence of the average critical path length on disorder, for arbitrary states s_{RND} (circles) and ground state s_{GS} (diamonds) as a function of R^2 . System analyzed for $N = 500$, $N_{\text{iter}} = 10$, $N_{\text{dis}} = 10$.

distance with respect to the H-states and merits further investigations.

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