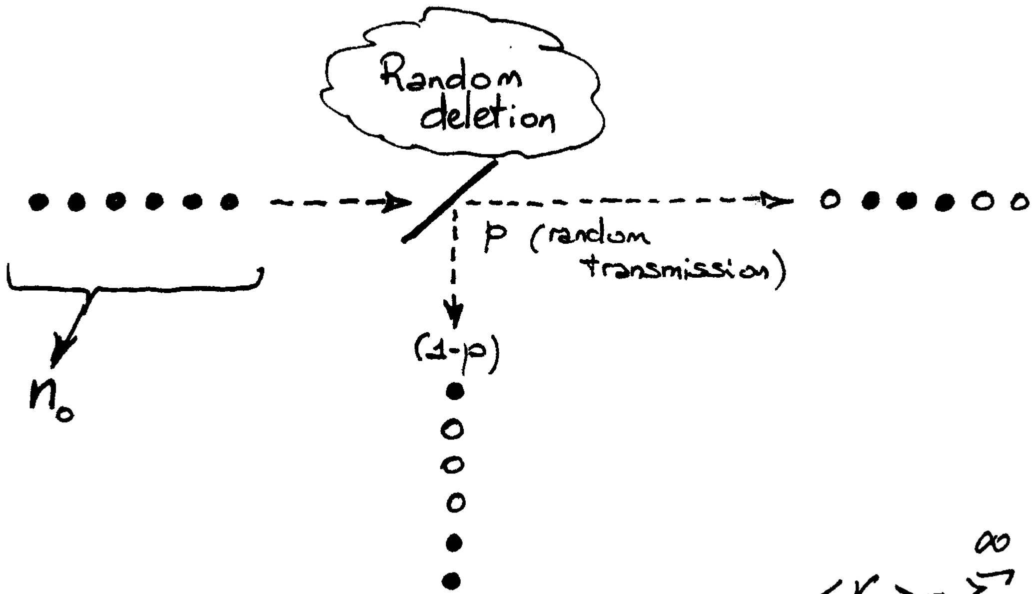
Partition noise (Burgess Variance Theorem)



$$P_K(K_0) = \sum_{n_0=K_0}^{\infty} \overbrace{P_n(n_0)}^{\text{initial distribution}} \cdot \overbrace{P_K(n_0, K_0)}^{\text{Bernoulli distribution}}$$

$$= \sum_{n_0=K_0}^{\infty} P_n(n_0) \cdot \binom{n_0}{K_0} p^{K_0} (1-p)^{K_0}$$

mean

$$\langle K_0 \rangle = \sum_{K_0=0}^{\infty} K_0 P_K(K_0) = \sum_{n_0=0}^{\infty} \sum_{K_0=0}^{n_0} K_0 \, P_n(n_0) \, P_K(n_0, K_0)$$

$$\sum_{K_0} K_0 P_K(n_0, K_0) = p n_0 \quad \text{(Bernoulli)}$$

$$= \sum_{n_0=0}^{\infty} (p n_0) \cdot P_n(n_0) = p \sum_{n_0=0}^{\infty} n_0 P_n(n_0) = p \langle n_0 \rangle$$

Variance

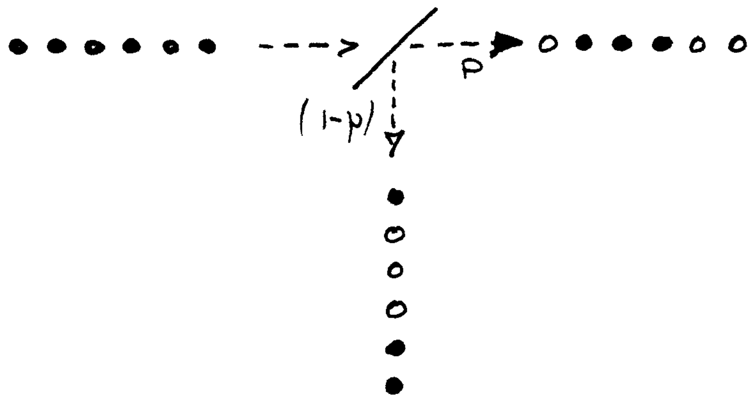
$$\langle K_0^2 \rangle = \sum_{K_0=0}^{\infty} K_0^2 \, P_K(K_0) = \sum_{n_0=0}^{\infty} \sum_{K_0=0}^{n_0} K_0^2 \, P_n(n_0) \, P_K(n_0, K_0) = \sum_{n_0=0}^{\infty} \left((p n_0)^2 + n_0 p (1-p) \right) P_n(n_0) -$$

$$\sum_{K_0} K_0^2 P_K(n_0, K_0) = (p n_0)^2 + n_0 \cdot p (1-p)$$

$$= p^2 \langle n_0^2 \rangle + p(1-p) \cdot \langle n_0 \rangle$$

Source

P_{n_0}, $\langle n_0 \rangle$, $\langle n_0^2 \rangle$



$$\langle K_0 \rangle = p \cdot \langle n_0 \rangle$$

$$\langle K_0^2 \rangle = \underbrace{p^2}_{\text{attenuation factor}} \underbrace{\langle n_0^2 \rangle}_{\text{initial variance}} + \underbrace{p \cdot (1-p) \langle n \rangle}_{\text{partition noise}}$$

If the initial distribution is a Poisson distribution, the $P(K_0)$ distribution is poissonian with any arbitrary deletion value and the varience is always equal to the mean value, i.e $\sigma_{K_0}^2 = \langle K_0 \rangle$. When the initial distribution has a larger or smaller variance with respect to a Poisson distribution, the Poisson limit ($\sigma_{K_0}^2 = \langle K_0 \rangle$) is obtained only at a very large deletion limit.

Burgess variance theorem $\quad \langle n^2 \rangle - \langle n \rangle^2 = \sigma_n^2$

$$\boxed{\sigma_{K_0}^2 = \sigma_n^2 \cdot p^2 + \langle n \rangle \cdot p(1-p)}$$

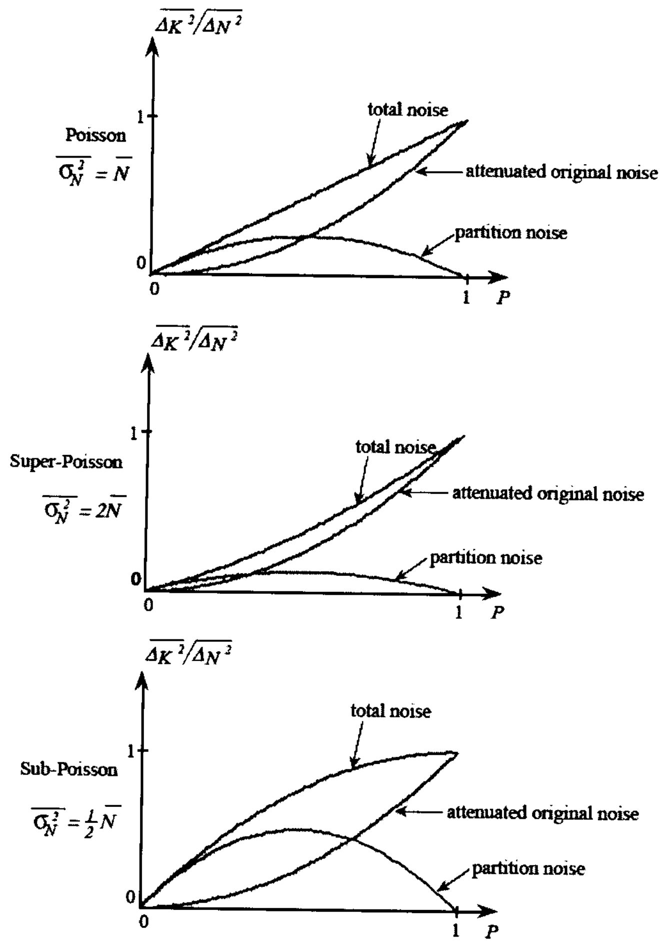


Figure 1.5: The change of the variance for a Poisson ($\sigma_N^2 = \overline{N}$), super-Poisson ($\sigma_N^2 = 2\overline{N}$) and sub-Poisson ($\sigma_N^2 = \frac{1}{2}\overline{N}$) distributions due to a partition process.